TEST I

Math 26L-04
February 11, 1998

Name: $\underbrace{\hspace{6cm}}_{\text{by writing my name i swear by the honor code}}$

Read all of the following information before starting the exam:

- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Justify your answers algebraically whenever possible to ensure full credit. When you do use your calculator, sketch all relevant graphs and explain all relevant mathematics.
- Circle or otherwise indicate your final answers, when applicable. Put a circle on the scrap page at the end of the exam for two points.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- This test has 10 problems and is worth 100 points, plus some extra credit at the end. It is your responsibility to make sure that you have all of the pages!
- Good luck!

TRIGONOMETRY "CHEAT SHEET"

You may not need all of the identities in this list, and you may need identities not in this list. Please note that you can derive many other trigonometric identities from the ones in this list.

$$\sin^2(x) + \cos^2(x) = 1$$

$$\sin(u+v) = \sin u \cos v + \cos u \sin v$$

$$\cos(u+v) = \cos u \cos v - \sin u \sin v$$

$$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

$$\sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-\cos x}{2}}$$

$$\cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1+\cos x}{2}}$$

$$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$$

$$\lim_{\theta \to 0} \frac{\cos\theta - 1}{\theta} = 0$$

1. (*10 points*) Suppose $\dfrac{dy}{dt} = \dfrac{2}{t}$ and $y(1) = 0$. Use Euler's method to approximate $y(2)$, using a time step of $\Delta t = \frac{1}{2}$. Recall that Euler's formula is $y_k = y_{k-1} + y'_{k-1}\Delta t$. Please show your work clearly and in order.

2. (*10 points*) Is $y(x) = e^x \ln x$ a solution to the initial value problem $\dfrac{dy}{dx} = \dfrac{e^x}{x} + y, \quad y(1) = 0$? Justify your answer.

3. *(12 points)* Consider the following table giving the length of day (*i.e.* number of hours of daylight) on various days of the year in Durham, North Carolina:

Date	Day	Hours of Daylight
01/18	18	10.0
03/15	74	12.3
05/26	146	15.3
06/19	170	15.7
07/29	210	14.8
09/07	250	13.3
11/26	330	9.9
12/20	354	9.5

Space for graphing and scratch work:

Find a function of the form $f(x) = A\sin[B(x - C)] + D$ that would be a good model for the data. Justify how you obtained each of the constants A, B, C, and D in the spaces provided. *(Hint: you do not need to use all of the data points! You may assume that the maximum and minimum day lengths are amongst the data above.)*

$f(x) =$ __________________________

(a) $A =$ ______ because:

(b) $B =$ ______ because:

(c) $C =$ ______ because:

(d) $D =$ ______ because:

4. (*10 points*) Prove that the derivative of $\arctan(x)$ is $\frac{1}{1+x^2}$. Be sure to show all work clearly and in order.

5. (*8 points*) Differentiate the following functions with respect to x.

(**a**) $h(x) = x\cos x + \tan x$

(**b**) $g(x) = e^{\arctan(3x^2)}$

6. (*12 points*) In the problems below, do all computations by hand; do not use your calculator. Justify your answers by showing all work clearly and in order. Include pictures of relevant angles, triangles, and the unit circle where appropriate.

(**a**) Find the exact value of $\cos(\frac{5\pi}{6})$.

(**b**) If $-\frac{\pi}{2} < t < 0$ and $\sin(t) = -\frac{2}{3}$, find $\tan(t)$.

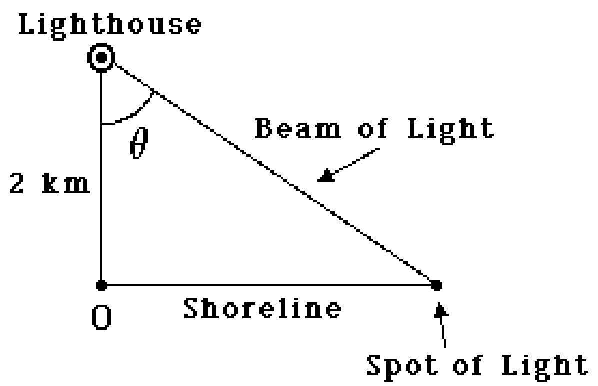
(**c**) Given that $\sin(\frac{\pi}{12}) = 0.259$ and $\cos(\frac{\pi}{5}) = 0.809$, compute $\sin(\frac{11\pi}{12})$.

7. (*10 points*) Newton formulated the principle that the rate of change of the temperature of an object is proportional to the difference between the object's temperature and the temperature of the surroundings. Suppose that a cup of coffee, which has temperature 110° F, is placed in a room which is at temperature 72° F. Suppose that we measure time in minutes after the placement of the coffee in the room and that the constant of proportionality is .7.

(**a**) Construct an initial value problem whose solution is the temperature $T(t)$ of the coffee at time t.

(**b**) Solve this IVP (show your work clearly please), and use the solution to find the temperature of the coffee after 2 minutes.

8. (*8 points*) A lighthouse is 2 km from the long, straight coastline shown in the figure below. Find the rate of change of the distance of the spot of light from the point O with respect to the angle θ.



9. (*8 points*) Label each of the items below as true (write TRUE) or false (write FALSE).

(a) ____________ $\sin(\frac{\pi}{2} - x) = \cos x$

(b) ____________ $\sin x = \sqrt{1 - \cos^2 x}$

(c) ____________ $\sec x$ is undefined at $x = \frac{\pi}{2}$

(d) ____________ $\tan(-x) = \tan x$

(e) ____________ $\cot x = \dfrac{\cos x}{\sin x}$

(f) ____________ $3 + \cos(2x)$ is an even function

(g) ____________ The range of $\cos^{-1}(x)$ is $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

(h) ____________ Elvis discovered the number "e".

10. (*12 points*) Solve the following initial value problems.

(**a**) $y'(x) = e^{-x} - 7x, \quad y(0) = 3$

(**b**) $\dfrac{dr}{ds} = 2s, \quad r = 4$ when $s = 1$

(**c**) $\dfrac{dr}{ds} = 2r, \quad r = 4$ when $s = 1$

(**d**) $\dfrac{dy}{dt} = \cos(3t) + 1, \quad y(0) = 1$

Survey Question (2 Extra Credit Points):

What did you think of this test? Were you prepared for the kinds of questions that were on this test?

Scrap Page

(please do not remove this page from the test packet)