Devil Round

December 19, 2006

1. The entrance fee the county fair is 64 cents. Unfortunately, you only have nickels and quarters so you cannot give them exact change. Furthermore, the attendent insists that he is only allowed to change in increments of six cents. What is the least number of coins you will have to pay?

2. At the county fair, there is a carnival game set up with a mouse and six cups layed out in a circle. The mouse starts at position A and every ten seconds the mouse has equal probability of jumping one cup clockwise or counter-clockwise. After a minute if the mouse has returned to position A, you win a giant chunk of cheese. What is the probability of winning the cheese?

3. A clown stops you and poses a riddle. How many ways can you distribute 21 identical balls into 3 different boxes, with at least 4 balls in the first box and at least 1 ball in the second box?

4. Watch out for the pig. How many sets S of positive integers are there such that the product of all the elements of the set is 125970?

5. A *good* word is a word consisting of two letters A, B such that there is never a letter B between any two A's. Find the number of good words with length 8.

6. Evaluate $\sqrt{2-\sqrt{2+\sqrt{2-\ldots}}}$ without looking.

7. There is nothing wrong with being odd. Of the first 2006 Fibonacci numbers ($F_1 = 1, F_2 = 1$), how many of them are even?

8. Let f be a function satisfying $f(x) + 2f(27 - x) = x$. Find $f(11)$.

9. Let A,B,C denote digits in decimal representation. Given that A is prime and A - B = 4, find (A,B,C) such that AAABBBC is a prime.

10. Given $\frac{x^2+y^2}{x^2-y^2} + \frac{x^2-y^2}{x^2+y^2} = k$, find$\frac{x^8+y^8}{x^8-y^8}$ in term of k.

11. Let $a_i \in \{-1, 0, 1\}$ for each i = 1, 2, 3, ..., 2007. Find the least possible value for $\sum_{i=1}^{2006} \sum_{j=i+1}^{2007} a_i a_j$

12. Find all integer solutions x to $x^2 + 615 = 2^n$ for any integer $n \geq 1$.

13. Suppose a parabola $y = x^2 - ax - 1$ intersects the coordinate axes at three points A, B,and C. The circumcircle of the triangle ABC intersects the $y - axis$ again at point $D = (0, t)$. Find the value of t.

14.