DUKE MATH MEET 2007: TIEBREAKER ROUND

The Tiebreaker Round is given to students who give an outstanding performance on the individual round. The students with the highest score in the individual round are invited to the front of the auditorium, where they will be seated in desks facing the audience. When time is called, each of the participants will be given the first question in the tiebreaker round. The participants will attempt to solve the tiebreaker round question individually. When the participant wishes to submit an answer, he or she will give it to any of the proctors. If the student has submitted the correct answer, he or she will be awarded 10 points for him or herself and will receive the next question of the tiebreaker round. If the student has submitted an incorrect answer, 1 point will be deducted from the student's score in the tiebreaker round and the student will have to continue attempting to solve that problem. Students are free to submit as many answers as they wish to any problem. The round will continue for 10 minutes or until all students have solved all three problems in the tiebreaker round. Rankings will be determined based first on the score on the individual round, followed by the score on the tiebreaker round, followed by number of correct solutions submitted, followed by time at which the last correct answer was submitted.

As all this is going on, all three problems will be projected at the front of the auditorium, so that the audience may attempt to solve the problems. The participants in the tiebreaker round are forbidden to turn around to look at the quetions, since all three questions are being projected. The audience is also forbidden to give hints or otherwise make comments regarding the problems.

Tiebreaker Round

1. Let $p_b(m)$ be the sum of digits of m when m is written in base b. (So, for example, $p_2(5) = 2$). Let $f(0) = 2007^{2007}$, and for $n \geq 0$ let $f(n+1) = p_7(f(n))$. What is $f(10^{10000})$?
2. Compute:
$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} 4n}{n^4 - 8n^2 + 4}.$$
3. $ABCDEFGH$ is an octagon whose eight interior angles all have the same measure. The lengths of the eight sides of this octagon are, in some order,
$$2, 2\sqrt{2}, 4, 4\sqrt{2}, 6, 7, 7, \text{ and } 8.$$
Find the area of $ABCDEFGH$.