Duke Math Meet 2011: Relay Round

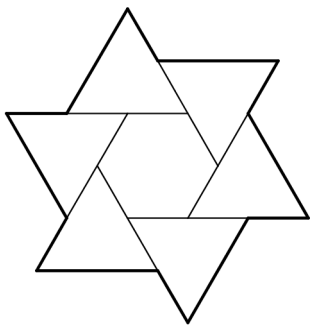
In the Relay Round each team of six students will divide into two groups of three students each. There are two sub-rounds in the Relay Round, each group of three students will work together. In each of the two sub-rounds, the three students in a group will receive a different problem. The problems that the second and third students in each group will have the symbol TNYWR within the problem statement. This stands for The Number You Will Receive. This is because the first student in the group is supposed to write down the answer to his or her problem and pass that answer to the second student, and similarly the second student is to pass the answer to his or her problem to the third students. The second student will need the first students answer to completely solve the second question, and similarly the third student will need the second students answer to completely solve the third question. The first and second students can only pass numbers that are fully simplified to the second and third students; there are to be no stray symbols, algebraic expressions, or other marks. The one exception is that students can underline numbers to indicate, for example, whether their answer is a 6 or a 9.

The first and second students may pass as many answers as they wish; however, the third student can only submit answers after three minutes or six minutes, and only the third studentss answer will be graded. If a group submits an answer after six minutes then their answer after three minutes, if they gave one, will be discarded. If a group obtains the correct answer after three minutes, they will earn 4 points for their team. If a group obtains the correct answer after six minutes, they will earn 2 points for their team. Therefore, a group should not, for example, submit the same answer after six minutes that they did after three minutes. The moderator will give 15-second warnings before the three minute deadline and the six minute deadline.

First Relay Round

1A. Find the number of positive integers n such that $-n^2 + 8n + 13 \geq 0$.

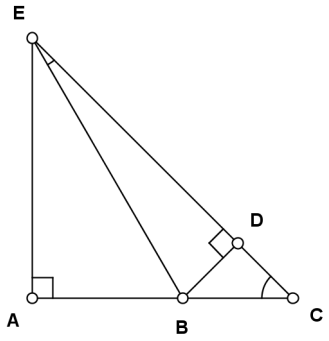
1B. Let $L = |6 - \text{TNYWR}|$. In the diagram below, six equilateral triangles are arranged in a ring such that a regular hexagon is formed in the middle. Each side length of the hexagon is half the side length of each triangle. Given that the perimeter of the hexagon is L, what is the perimeter of the exterior of the diagram?



1C. Let $L = \lfloor \text{TNYWR} + 1 \rfloor$. In the written language Flipsy, there are only two characters: $\vee$ and $\wedge$. A word in Flipsy is a sequence of characters. A word is "valid" if it reads the same when it is rotated 180 degrees as it does when it isn't rotated. For instance, $\vee\wedge$ is valid, but $\vee\vee$ isn't. How many valid words with L characters are there? Note that $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x.

Second Relay Round

2A. In the figure below, $AE = 4\sqrt{3}$, $\angle EAC$ and $\angle EDB$ are right, $\angle BED = 15^\circ$, and $\angle ECA = 45^\circ$. What is the area of $\triangle EBD$?



2B. Let $L =$ TNYWR. Find the number of integers n between L and $4L$ such that

$$\sqrt{n} - \lfloor \sqrt{n} \rfloor < 1/2.$$

Note that $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x.

2C. Let $L =$ TNYWR. Right triangle T has integer side lengths, one of which is L. What is the second-largest possible value of the perimeter of T?