Theme 5. Implementation

In the six papers in this section, we learn from the experience of others who have implemented changes at their institutions, who have developed new curricular materials and designed new courses, or who are utilizing emerging technologies. We also revisit the impact of education reform on the transition from high school to college and the appropriate placement of students. In particular, Robert Megginson offers some suggestions for successfully implementing a new curriculum from the faculty standpoint, while Judy Ackerman offers suggestions for successfully implementing a new curriculum from an administrator's standpoint. In support of Zalman Usiskin's earlier claims about the importance of placement, Sheldon Gordon argues that implementing new pedagogies and new curricula necessitates rethinking the way students are placed into college courses. Lawrence Moore and David Smith discuss the impact of technology on the way students learn. Al Cuoco describes designing new instructional materials that are based on students' "habits of mind," instead of being topic driven. Many participants at the conference, *Rethinking the Preparation for Calculus*, felt that the problem with college algebra and precalculus is that they are trying to serve too many audiences. Bonnie Gold describes how at her institution, they solved the one-course-does-not-fit-all situation by dividing their college algebra course into several different courses "each with a clear mission and a separate clientele."

The University of Michigan at Ann Arbor initiated some major curricular revisions in its precalculus and introductory calculus courses in the early 1990s.... The changeover from more traditional to reformed courses in the first-year program required an intensive and, at times, exhausting effort, but actually went remarkably smoothly considering the size of the undertaking and the controversies that reform efforts elsewhere have sometimes faced. We did encounter a number of practical and political issues along the way that had to be dealt with to assure a successful transition, some of which we anticipated and others which caused us to scramble a bit when they arose. The purpose of this paper is to describe some of those issues and provide suggestions for handling them.

Some Political and Practical Issues in Implementing Reform Robert E. Megginson

It is easier to effect change in precalculus courses if the dean is on board than without the dean. The dean can be an ally when faculty recognize the need for change, or the dean can be the instigator when the mathematics department resists change. Be prepared to educate your dean about the issue and make sure that you understand the current state of affairs with respect to students in precalculus at your college. In this way there is a chance that you can make an effective case for how a reformed precalculus course will improve things for students.

Implementing Curricular Change in Precalculus: A Dean's Perspective Judy E. Ackerman

In large measure, the problems with mathematical transitions are due to the rapidly growing reform movements in mathematics education at both the secondary level and the college level. NCTM's efforts to promote a school curriculum based on their *Standards* documents are bearing fruit around the country. Instead of the relatively uniform secondary curriculum that most of us went through, many schools across the country have implemented a variety of reform curricula that provide students with very different content and very different teaching and learning environments.... The smooth transition from school to college mathematics is breaking down.... However, the transition problems involve considerably more than differences between school and college mathematics offerings. Perhaps the most significant, yet often overlooked, aspect of transition is the issue of placement—the interface between the two.

The Need to Rethink Placement in Mathematics Sheldon. P. Gordon

Students with notebook computers connected to a campus backbone by wireless cards are increasingly common. Extensive use of communication technology such as NetMeeting is less common but should be the norm in a couple of years. And, if this were an interactive, online article, we could provide a live link to video of students working through our module. If our scenario is an accurate glimpse of the future—and we believe this future is almost upon us—what are the issues for student learning? ... Technology is changing the way students approach learning. Increasingly, they will conceive of their work in terms of interactive learning materials, computer algebra systems, spreadsheets, and Web-based cooperation—with occasional use of pencil and paper. Learning how to learn in this environment is as important as learning about the mathematics itself.

Changing Technology Implies Changing Pedagogy Lawrence C. Moore and David A. Smith

Some very useful "modes of thought" in mathematics are given short shrift in high schools (and especially in precalculus courses): hardly showing up at all are reasoning about algorithms, combinatorial thinking, and using the linearity of certain maps on the plane. Furthermore, even for students who go on to calculus and advanced mathematics, the emphasis on traditional precalculus skills and methods is misplaced. Calculus instructors have long complained that the real stumbling blocks for their students are the hard *ideas* in the subject: notions like limit, approximation, convergence, and error estimation. Organizing curricula around these *mathematical habits of mind* provides an alternative to topic-driven design.

Preparing for Calculus and Beyond: Some Curricular Design Issues Al Cuoco

A cornerstone of the American democracy is that all children should be given equal opportunity. As a result, the standard school mathematics track leads to calculus. While this may be a reasonable policy at the school level, by the time students arrive at college, they have become unequal in many ways. Some have been stimulated by their school mathematics, while others have been crippled by their early mathematical experiences. Some have a clear interest in a mathematically intensive discipline, while others are clearly focused on the humanities, business or social sciences and others are still undecided. One size no longer fits all (if it ever did), in college mathematics courses.

Alternatives to the One-Size-Fits-All Precalculus/College Algebra Course Bonnie Gold

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Some Political and Practical Issues in Implementing Reform

Robert E. Megginson

Mathematical Sciences Research Institute

The University of Michigan at Ann Arbor initiated some major curricular revisions in its precalculus and introductory calculus course in the early 1990s with which the author has been closely involved as a member of the Michigan mathematics faculty. A number of political and practical issues had to be addressed to help assure the success of the efforts. The purpose of this paper is to describe some of those issues and provide suggestions for dealing with them when they arise in other implementations.

Introduction

In the fall semester of 2001, about 3000 students enrolled in the three courses considered to be part of the University of Michigan's first-year mathematics program, namely, differential calculus, integral calculus, and Michigan's one precalculus course. About 700 of those students were in precalculus, most of whom were taking the course specifically to get ready for courses in calculus for which placement information indicated they were not yet fully prepared. All three of these introductory courses are taught by methods commonly called "reformed" featuring the appropriate use of technology, texts [1], [3] that support the pedagogical emphasis in the courses, and various forms of cooperative learning and other teaching methods not based exclusively on lecture to take advantage of different student learning styles.

These curricular reforms, in essentially their current shapes, have been in place since 1992 in the case of calculus and 1993 for precalculus, with the precalculus reform following hard on the heels of that for calculus so students would not experience a sudden change in the look and feel of the courses when passing from precalculus to the first calculus course.

The changeover from more traditional to reformed courses in the first-year program required an intensive and, at times, exhausting effort, but actually went remarkably smoothly considering the size of the undertaking and the controversies that reform efforts elsewhere have sometimes faced. We did encounter a number of practical and political issues along the way that had to be dealt with to assure a successful transition, some of which we anticipated and others which caused us to scramble a bit when they arose. The purpose of this paper is to describe some of those issues and provide suggestions for handling them. Though this paper appears in a volume on curricular changes in precalculus, in practice the same issues can arise in any effort to reform introductory mathematics courses, and so are addressed here in that more general context. The first is one that can quickly doom a nascent reform effort if colleagues get the idea that *they* are considered to be the biggest problem that needs to be addressed.

Show respect for your colleagues' teaching styles

At all costs, one must avoid sending the message to colleagues that those involved in a reform effort have found the secret to good teaching, and those who do not use the methods must therefore be bad teachers. Frankly, some of the rhetoric from curricular innovators in the early days of reform sent this message quite loudly. Faculty who have been caring teachers doing an excellent job using traditional lecture-oriented methods, but often with some truly innovative twists to those methods, quite rightly resent the implication. Because of this, whenever I give a talk about the Michigan reformed precalculus and calculus programs at another institution, it almost always happens that someone in the audience asks why "we" (those of us who have been involved in reform projects) believe "they" are all bad teachers. "We" certainly do not—at least those of us do not who remember how much we learned about teaching from watching some superb teachers who use traditional methods—and we need to say so. A reform effort will not succeed without support from our colleagues, and we cannot expect to have that if we do not respect what they have accomplished in the classroom, even if we believe the pedagogical methods associated with reform will often be more effective with more students.

More generally, implementers of a mathematics reform project should do everything possible to avoid creating an "us" versus "them" division in the department over the project, and this means listening closely to colleagues' concerns about the effect of the curricular revisions and addressing them where possible. To be able to do this, one must make sure colleagues actually know what is changing and why.

Keep your colleagues in the loop

Make certain that colleagues understand from the beginning the full extent of your reform effort, and how its pieces fit together. In particular, make sure they understand that reform is not just the selection of the textbook, or the use of technology. In part, this is to assure that your colleagues do not feel blindsided when they later see that the changes went beyond a new textbook or the introduction of calculators into a course. You may also find that there will be more support for your effort if you can demonstrate how its pieces complement each other, with the textbook, technology, and pedagogical changes working together to enhance student learning. A seminar or two about your intentions and the problems that will be addressed, with some hands-on work with a few difficult and interesting exercises from the textbook you will be using, can do much to reassure your colleagues about the likely results of the curricular changes being planned. It is particularly important to make sure that the persons most likely to be resistant to the changes attend these sessions; invite them personally.

Get the backing of senior faculty

It is important to have senior faculty who are part of the power structure of the department buy into your reform effort at a very early stage, preferably by taking a direct part in it. Our precalculus and calculus reform efforts were aided greatly by the strong support of the chairs of our department, D.J. Lewis and B.A. Taylor, during the implementation phase, and administrative support for the programs has remained strong since then. The calculus reform effort that preceded and laid the groundwork for our precalculus reform was directed by a respected senior faculty member, Morton Brown, with the help of another faculty member, Patricia Shure, who is well known at the national level for instructor preparation and educational innovation. Though most of the rest of our faculty in the early 1990s were not really familiar with the issues that calculus and precalculus reform were addressing, most did know that there was already some controversy surrounding reform. However, with senior departmental personnel supporting the reform efforts, the rest of the faculty were willing to give the curricular revisions a chance to prove themselves, and in many cases to teach the revised calculus courses to see for themselves how they had changed. Unfortunately, but not unexpectedly, senior faculty involvement in teaching the reformed courses

has been almost exclusively in calculus, with only three having taught precalculus at the time of this writing¹. However, the reforms in the precalculus course are very similar to what we did with calculus, and the faculty know that. In any case, almost all faculty who have actually taught the courses are now convinced of the value of the changes. With broad, continuing support from departmental administration and faculty, the curricular changes are now institutionalized.

Because Michigan's reform programs are well known nationally, graduate students and postdoctoral faculty who have taught in the programs are sometimes sought by other departments wishing to implement their own reform programs. A job candidate in this situation needs to check out carefully where the support for reform in that department actually lies. If there is a strong commitment from the departmental administration and a substantial collection of senior faculty for a change in the courses, and a solid understanding of the issues that reform is supposed to address, then well and good. On the other hand, if that department seems not to be sure why they might want to reform their courses in the first place, but wishes to try an experiment by bringing in someone from outside to conduct a few sections of a reformed course and see what happens, then there is a great potential for professional disaster for the job applicant. Junior faculty already in a department who wish to undertake a substantial reform effort without obtaining the backing of respected senior faculty, but instead assume that the changes will automatically prove themselves, should have similar concerns.

Get the backing of client departments

Client departments should be brought on board from the beginning. They can be a great source of support with the higher administration, as well as within your own department when faculty from the client disciplines can help reassure uneasy faculty from your own that the changes in the courses should have a positive impact on preparing students for study in other mathematically-based fields. It may turn out that the client departments are concerned with the same pedagogical issues that you wish to address, and will be quite supportive if they understand what it is you are doing and why you are doing it.

At the beginning of our reform efforts at Michigan, we had extensive meetings with the science and engineering departments about our intentions and to get their advice on how our revisions could better prepare students for courses in those departments. The result has been generally strong support by those departments for the program and its goals. One piece of anecdotal evidence of this occurred in a joint meeting of the curriculum committees of our liberal arts and engineering colleges attended by the author of this paper. A faculty member who is not in one of our usual client disciplines, but who had read an anti-reform article in the popular press, initiated a discussion about whether our reform efforts had compromised Michigan's introductory mathematics program. The most vocal supporter of our efforts in that meeting turned out to be a physics professor who said that, based on the early discussions between mathematics and physics about the goals of the reform programs, his department had decided to try assigning exercises in the introductory calculus-based physics course that would require students not just to be able to compute integrals mechanically, but to understand more conceptually what integrals really represent. It was discovered that the students could actually do those exercises, which our supporter from physics was confident would not have been the case prior to our reform efforts. He closed by stating that he would not want mathematics to go back to our previous way of presenting the material. His words in that meeting had a far more positive impact than just about anything a member of our own department could have said to defend our program.

Get the backing of academic counselors

It is important to explain to your institution's academic counselors the reasons for the reform and how it will affect their advising of students, and obtain their support for the program. Many students taking

¹Editor's note: The original version of this paper was submitted in February, 2002.

college precalculus have already done well in high school mathematics; about a quarter of our precalculus students in the fall 2001 term had previously taken a high school *calculus* course. When faced with a reformed course as an introduction to college mathematics, such a student may become very concerned that someone has changed the rules under which the student had done well in the past, particularly after a bad first quiz or homework grade.

When this happens, the student is likely to head straight for an academic counselor for advice. It is important that the counselor be able to explain to the student the reasons for the curricular revisions and their ultimate benefits, and provide suggestions for improving performance that might be a bit different from those appropriate for a more traditional course. The advisors will be able to do this if they have been brought into the loop early in the planning stages of the project and have had any concerns of their own about the curricular changes addressed, and in this case they can end up being some of your strongest supporters. If this is not done, then the advisors will be at a loss about what to tell students who are in trouble in the course. They also might end up advising students away from it at registration time, and spreading the word on campus that there seems to be something strange going on in your introductory program.

The next recommendation addresses one very reasonable concern that colleagues in your own department are almost certain to bring up, and that you might also hear from colleagues from other departments and academic counselors if the issue is not dealt with from the beginning.

Make sure skills are learned

In the early days of the current mathematics reform efforts, quite a bit was said about the need to increase the emphasis on concepts and understanding in introductory college mathematics courses, with correspondingly less stress placed on algebraic and computational skills. Though the de-emphasis on skills was in most cases never as great as some of the rhetoric on both sides of the reform issue would have one believe, this issue became a hot-button item for many persons worried about the early direction of mathematics reform. The concern by colleagues that students might not learn needed skills in courses that are supposed to prepare them for more advanced study has brought down fledgling reform efforts in more than one department. To help allay fears about this issue at Michigan, and, more importantly, to make sure our students really were getting required skills from our precalculus and first-year calculus courses, we implemented gateway examination programs in those courses.

A gateway examination is a test of a student's mastery of important basic skills, such as applying fundamental differentiation and integration rules quickly and accurately, that need to be part of a student's personal mathematical toolkit even though computer algebra software or calculators can do the computations. In most implementations of gateway testing, including ours, students may continue taking different versions of a gateway examination over a particular set of skills without penalty so long as the test is finally passed by some deadline, which allows the student to shore up shaky skills between attempts. However, in trade for being allowed the multiple attempts, the skill level required to pass is high. For example, on Michigan's eight-question differentiation test containing some quite difficult derivatives covering all of the basic differentiation rules, the student is allowed to miss only one question, and errors the student might think are small, such as an omitted set of parentheses, are not forgiven.

The effort involved in implementing and maintaining a major gateway-testing program should not be underestimated. So that a student will get fundamentally different versions of a test on successive attempts, Michigan's gateway tests have been computer-generated from the beginning of the program in the early 1990s. However, before the 2001–02 academic year most of the tests were given in paper form, mostly in a testing center rather than in the classroom, and were proctored and graded primarily by undergraduates; (see [4] for a description of the early days of the program). The logistics involved in administering, grading, and returning thousands of tests in a timely fashion each semester were formidable, so almost from the

beginning we sought ways to mechanize as much of the process as possible. By the late 1990s, computerbased testing systems were becoming sophisticated and stable enough to handle our requirements, and at the time of this writing Michigan is in the midst of a two-year effort to convert all of these tests to be administered in a computer laboratory designed specifically for this purpose, using testing software originally developed by John Orr and collaborators at the University of Nebraska and currently marketed by John Wiley & Sons under the name eGrade. This conversion and a further extension of the program has been made possible by the support of the National Science Foundation, through grant DUE-0088264.

Know who takes the courses

It can happen that a successful program at one institution will not work well at another because of fundamental differences in the student populations. When problems arise because of a bad fit between program and students, it might be possible to make matters better with a few modifications, but it might also be too late if the project is already perceived to be a failure. For this reason, it is particularly important for implementers of reformed introductory mathematics courses to consider the nature of the population to be addressed by the reforms.

At the University of Michigan, the "typical" precalculus student is a first-year student in the liberal arts college, with the goal of preparing for calculus rather than taking a liberal arts mathematics course, and has not ruled out the possibility of a major or minor in mathematics. Here is one example of how such information affects the conduct of the course. A major feature of the program is group homework, with extensive exercise sets due weekly from students who solve the exercises together outside of class in teams of size three or four. We would have been more concerned about the logistical issues students would face in arranging meetings to do the group homework if we did not know that a sizable majority of them, as first-year students, live in residence halls that are located in four clusters on campus. Where possible, the initial assignment of students to homework groups is made so that the students in each group live near each other. If one were to attempt to transplant the Michigan model without modification to another institution where most of those taking precalculus were nontraditional students living at home, then scheduling meetings outside of class to work on group homework could cause major problems.

Prepare instructors for a changed classroom

At Michigan, there is a weeklong professional development program before the fall term starts that is required of all instructors who are going to be teaching the reformed courses for the first time. This is followed up with weekly meetings in precalculus and differential calculus where further pedagogical issues are addressed, often as they arise in the classrooms. Visits are also made to the classroom of each instructor new to the program, usually twice during the term.

This instructor training model may not be practical for institutions where only a few instructors would require the training each year, but it is still important for those instructors to learn what they will need to do in a classroom that may be radically different from those in which they learned mathematics. One good resource for such instructors, whether or not a full-blown instructional training program is available to them, is [2]. Both authors of that volume are former Michigan instructors who helped with the coordination of the reformed courses and instructor development program.

Finally, and perhaps most important ...

Do not underestimate the total impact that a curricular change can have on the department. Because teaching precalculus and calculus is such a large part of the role of almost every mathematics department, a serious curricular change in the first-year courses will affect the entire operation of the department. If a

commitment to smaller class sizes is made, then there is an obvious impact on the hiring of faculty and the support of teaching assistants. Reformed courses taught in multiple sections tend to require more attention from a course coordinator than more traditional courses, particularly when the instructors have not taught such courses before, in which case the reward system in the department may need some modification to assure that the coordinators are appropriately rewarded, both monetarily and professionally, for their efforts. When many instructors will be teaching the courses, training the instructors and following up with classroom visits can consume substantial resources. All of these issues, as well as others specific to the implementation, will quite likely require an increase in resources for the department as a whole and a reprioritization of resources within the department.

The personal effort required from someone coordinating part of a reformed introductory mathematics program can also be substantial. There are occasions on which each of us involved in the Michigan program would go home quite exhausted, or occasionally would not go home at all; more than once I watched the sun set from my office window while working on some problem involving the coordination of precalculus or calculus, and then saw it rise again before leaving. However, the effort is worth it. Michigan's students are now getting better courses from instructors who are better prepared to address differing student learning styles, and that is paying dividends for both the institution's own programs and those of the institutions that Michigan's teaching assistants and postdoctoral faculty ultimately make their academic homes.

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Implementing Curricular Change in Precalculus: A Dean's Perspective

Judy E. Ackerman Montgomery College

Introduction

Mathematics departments have not been overly enthusiastic about rethinking precalculus courses despite changes in calculus, changes in K-12 mathematics that have resulted from the NCTM Standards, and an increased emphasis on accountability. In four-year colleges and universities, some faculty equate precalculus with precollege mathematics or at best as the one mathematics course that students take to meet their graduation requirement. However, in the two-year colleges, precalculus often serves as a true pathway to calculus and to majors that require a significant amount of mathematics.

For many years, calculus reform was the rallying point for mathematics faculty around the country. It involved much more than the addition or deletion of topics from the calculus curriculum. Rather, it initiated the fundamental questioning of what was really important for calculus students to know, particularly in the light of the increasing availability of technology in the form of computers, graphing calculators, and computer algebra systems. The balance of depth versus breadth, applications, and theory was questioned. Calculus reform was much more than just curriculum reform since it also demanded significant change in pedagogy and assessment. Today, even so-called "traditional" mathematics courses and textbooks reflect elements directly attributable to calculus reform. With the history of collegiate calculus reform, why isn't precalculus reform being embraced by the mathematics faculty?

Initiating curricular change

What will it take for significant change to take place in the collegiate precalculus course? Who needs to get on board for it to happen? Although the literature is relatively silent about a dean or administrator's role in curricular reform there are a few suggestions that indeed there is a role and informed deans can be advocates for change. In *Crossroads in Mathematics: Standards for Introductory College Mathematics Before Calculus* it is suggested that although the faculty have the primary responsibility for implementing educational reform, deans can facilitate reform by providing leadership, resources and incentives [1].

A few years ago, when I was the chair of a mathematics department in which there was a limited amount of interest in implementing curricular change in some of the courses, I handed my dean an article by Lynn Steen. Steen articulated twenty questions that deans should ask their mathematics department if they wanted to improve mathematics instruction on their campus [7]. Since the dean had not seen the article,

this was a way to initiate a long overdue dialogue between the dean and the mathematics department in order to accomplish change that would benefit our students. Steen's questions were independent of course, pedagogy, technology, or type of higher education institution, and are still applicable. Today, as we grapple with the issue of fundamental change in precalculus, the following questions based on Steen's earlier questions might be particularly relevant to the discussion:

- Who are the students at your institution and what mathematics preparation do they come with?
- What do your students achieve in your precalculus course and in each of your other mathematics courses?
- Do you know what happens to students after they leave your precalculus course?
- Is technology used extensively and effectively in mathematics courses?
- Are the mathematics faculty aware of the national discussion concerning the NCTM Standards, AMATYC Crossroads, Quantitative Literacy, and MAA's work on the first undergraduate mathematics course?
- What steps has your department taken to be sure that faculty are well-informed about curriculum studies and research on how students learn?
- What resources are required to achieve the objectives that will result in change in your precalculus course?
- How well do department priorities match institutional priorities?

Of course, since each institution is different, the answers to these questions will differ and should inform how each college addresses the issue of change in precalculus.

Deans should not sit around indefinitely waiting for mathematics departments to initiate needed improvements in their courses. There are a number of trigger points that should signal to a dean that the mathematics department needs to take a close look at what is going on in their precalculus course. If the department doesn't raise the issue of change, then the dean should raise it when one or more of these is present:

- The success rate in precalculus is significantly lower than for other introductory college level mathematics courses.
- The success rate in calculus I for students who complete precalculus at the institution is low.
- The number of students who successfully complete precalculus and go on to calculus I is small.
- Departments that offer courses with a prerequisite of precalculus are complaining about students' mathematics preparation for these courses.
- The pattern of student complaints about precalculus is different than for other introductory college-level mathematics courses.

One of the issues regarding precalculus reform is that there is not a well-defined definition as to what is meant by precalculus. In fact, at the national workshop held in October 2001, Rethinking the Preparation for Calculus, participants were talking about precalculus with a big "P" being different from precalculus with a small "p." An additional source of confusion comes about because in some colleges, college algebra is the precalculus course. Since it's pretty clear that those of us in the mathematics community have some difficulty defining what is meant by a precalculus course, how can we expect those from outside of the mathematics community to understand the distinction between "Precalculus," "precalculus," and "college algebra" and advise students appropriately? So, to clarify discussion on precalculus course reform, I recommend that we come up with better names for these courses that clarify the intent of each of them. Then we can proceed on the task of reforming all three of these so called precalculus courses.

One of the courses, that today is often called "Precalculus," is for students who plan to continue on through a rigorous calculus sequence. The name "Precalculus" might even be reserved for this course.

Another course is for students who expect to take a limited number of additional mathematics courses that might include applied calculus and/or statistics. Today such a course might be known as either "precalculus" or "college algebra." Finally there is the mathematics course that is frequently called "college algebra," that students take as their last college mathematics course. There are usually administrative policies that require this course to be called college algebra, but many different types of courses come under this name. For example, in Maryland, the Maryland Higher Education Commission (MHEC) initially planned to issue regulations defining the statewide general education requirement in mathematics for all two-year and four-year college graduates as college algebra. Mathematics faculty from around the state's two-year and four-year public colleges got together and proposed a modification of the wording to "at or above the level of college algebra." Although this policy is not totally problem-free, introductory college level mathematics courses can be identified with a meaningful name that describes the actual scope of the course.

Let's start with the assumption that the purpose of the Precalculus course is to prepare students for a calculus I course. Does it make sense today, to offer the same type of manipulative-oriented, skills-driven, precalculus course that was offered in the past in which we assumed that most of the students in the class were going to be math majors or majors that required a significant amount of mathematics? Instead, shouldn't we be considering changes in pedagogy, content, type of applications and use of technology that are consistent with changes that have already been made in most calculus courses and in the K–12 preparation of the students who come directly from high school? This same argument can also be made for precalculus and college algebra courses.

Implementing curricular change

Case studies on curricular change in higher education are few and usually relate to changes in general education programs. Sandra Kanter, Director of the Doctoral Program in Higher Education Reform at the University of Massachusetts–Boston, suggests that change not be viewed as a one-time occurrence, but rather as a series of incremental happenings. She further asserts that "successful implementation of curricular changes required the energies and talents of many faculty members. To the degree that the process was open and collaborative, it built trust and good will among and between faculty and administrators, and only this ensured that faculty felt committed to the eventual outcome" [3].

An article by Alison Schneider [6], contrasting what happened in the overhaul of general education at two universities, points to the need for political savvy, considerable time spent in anticipating objections, and the active participation of the dean. At the university where the dean promised to provide the resources necessary to implement the new plan the revamped curriculum succeeded, whereas it did not at the other university.

Robert Diamond suggests that in many institutions of higher education the faculty promotion, tenure and reward system doesn't recognize significant time and energy devoted to improving courses and curricula [2]. If this is the case, there is actually a disincentive for faculty to make changes to precalculus courses or any other course.

The Long Island Consortium for Interconnected Learning reported in its progress report for year one [4] and year two [5] on how one of the deans from a member institution said that evidence of instructional innovation would be required for promotion to full professor in his college. This is an example of a dean taking on a leadership role and providing incentive for faculty participation in instructional innovation and curricular reform.

Making a case to the dean

Faculty who hope to initiate change in one of the precalculus courses at their college cannot assume that their dean is familiar with the issues surrounding the course. They need to be prepared to make a reasoned

argument that makes the case as to how students at their college will benefit from the proposed change(s). A clear understanding of who enrolls in precalculus and what they take next is crucial. One of the most powerful arguments to make to a dean is that a reformed precalculus course will increase student success and satisfaction. You should be prepared with knowledge of the current status at your college of the course and its outcomes for students, in addition to being acquainted with the existing literature that documents the need for this type of change in precalculus courses. Be ready to suggest new models for the course.

An implementation plan needs to include the goals to be accomplished by a reformed precalculus course, specific strategies to carry them out, and anticipated outcomes resulting from the implementation. It also needs an evaluation component so that the effectiveness of the change(s) can be documented. Don't forget that important outcomes may also be in the affective domain. Course objectives usually don't state that students completing a precalculus course will be interested in enrolling in an additional, non-required mathematics course, or that students completing a precalculus course will consider majoring in mathematics. In most other disciplines these are goals, so why shouldn't they be goals for mathematics courses too?

Finally, the dean needs to be given a realistic estimate of the resources necessary to implement a reformed precalculus course. Consider the resources needed to start the project as compared to those needed to maintain a project. It helps if you have considered alternative ways to fund the project. Are the proposed changes such that partial funding might be available from the National Science Foundation to develop a new course, or to adapt an existing, reformed precalculus course?

Carefully consider how much faculty development will be necessary to implement a reformed precalculus course. In every implementation of a reformed mathematics course I have been involved with, faculty development has been the underestimated component of implementation. Plan for time to prepare for implementation and to anticipate all of the potential difficulties that might be encountered. Request support for a project coordinator who can stay on top of the implementation process. Require each faculty member teaching the reformed course to participate in regular course meetings during their first semester teaching the course. Depending on the teaching load at your institution, you may want to provide alternate time for faculty for this activity too.

Pilot first

A pilot implementation of a reform precalculus course provides the opportunity to observe the intended and unintended effects of the reformed course and make necessary adjustments. For colleges with a large number of students and many different instructors, I would highly recommend this approach. This is also recommended when a mathematics department is not in general agreement as to the nature of a reform precalculus course.

The proposal for a pilot implementation of a reform precalculus should specify the length of time of the pilot, the number of sections to be included in the pilot, and the criteria to be used in deciding whether or not to go from the pilot to full implementation. For example, early on when our mathematics department piloted the use of graphing calculators in our precalculus course, there were those who were sure that students using graphing calculators would not perform well in calculus I. During the pilot we learned that students using graphing calculators did as well as those who did not use them even when they went on to take a traditional calculus I course. We learned other things as well including the fact that over half of our precalculus students did not take a calculus course with us during the two years following their successful completion of either version of the precalculus course. At the conclusion of the pilot, graphing calculators were required in all sections of precalculus.

Some of the criteria that you might consider to evaluate a pilot of a reformed precalculus course would be student success in the reformed precalculus course, change in student attitude towards mathematics, student success in their next mathematics course, and student enrollment patterns in an additional mathematics course. Although cost of a reformed course might not be a major consideration for faculty, this is something that deans need to consider. As a dean, I have chosen to go with an instructional choice that is somewhat more expensive when there has been a positive impact on student success. It is important to be open to changes or results that were not anticipated.

Support for faculty implementing change

Professional development is an important component of implementing curricular and instructional change, particularly for the faculty who did not initiate the project. After all, the faculty who support change are already knowledgeable about the change while the others who will be teaching the course need to be brought on board. In a department where the majority of the faculty who teach precalculus are adjuncts or TAs, this can be a problem. If incentives can be provided, this is the place. They may be in the form of travel to a conference, alternate time for project activities, or in the case of adjuncts, extra pay.

Although deans are not usually involved in the day-to-day implementation of curricular reform, it is a good idea for the project leadership to keep the dean informed about how things are going and to alert the dean to what unexpected things are happening. There should also be opportunities for informal discussions of the project between the dean and the rest of the project team. Finally, encourage and help the dean to understand the student perspective. The best way to do this might be to invite the dean to sit in on the course that is being changed. Whenever I sit in on a class, I always try to ask the students about the course at the end of the class period. The student perspective is important and may help shape a better course.

Summary

It is easier to effect change in precalculus courses if the dean is on board than without the dean. The dean can be an ally when faculty recognize the need for change, or the dean can be the instigator when the mathematics department resists change. Be prepared to educate your dean about the issue and make sure that you understand the current state of affairs with respect to students in precalculus at your college. In this way there is a chance that you can make an effective case for how a reformed precalculus course will improve things for students.

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The Need to Rethink Placement in Mathematics

Sheldon P. Gordon

Farmingdale State University of New York

Several years ago, Richard Riley, secretary of education in the Clinton administration, challenged the mathematics community to address the problems of articulation in mathematics education between high schools and two- and four-year colleges. Riley called for this national initiative, through the National Research Council, because of the growing breakdown in the once smooth transition between high school and college mathematics, as well as the differences between mathematical experiences in different colleges when students transfer from one institution to another.

In large measure, many of the problems with mathematical transitions are due to the rapidly growing reform movements in mathematics education, both at the secondary level and at the college level. NCTM's efforts to promote a school curriculum based on their *Standards* documents are bearing fruit around the country, as described in other articles in this volume. Instead of the relatively uniform secondary curriculum that most of us went through, many schools across the country have implemented a variety of reform curricula that provide students with very different content and very different teaching and learning environments.

- There is a major emphasis on conceptual understanding, not just routine manipulation;
- There is an emphasis on realistic problems, not just artificial template problems whose solutions are to be memorized and regurgitated;
- There is an emphasis on mathematics via discovery, not mathematics as a collection of facts and procedures to be memorized;
- There is an emphasis on the use of technology;
- There is an emphasis on writing and communication and working collaboratively.

Most of these themes are also part of the reform movements in collegiate mathematics. However, the extent to which these changes have permeated school mathematics is considerably more extensive than the extent to which they have affected collegiate mathematics.

Thus, the smooth transition from high school to college mathematics is breaking down. In particular, we have the following four scenarios:

- a traditional high school preparation leading to traditional college offerings.
- a traditional high school preparation leading to reform college offerings.
- a *Standards*-based high school preparation leading to traditional college offerings.
- a Standards-based high school preparation leading to reform college offerings.

The first of these scenarios should present no major *transition* problems, either to the students or to the institutions. Students are placed into courses offered in the same spirit as their high school experiences and the level of the courses should be comparable to the students' level of previous accomplishment. The fourth scenario should likewise present no major transition problems. (Of course, students can still encounter significant mathematical problems, but that is another issue altogether.)

However, the second and third scenarios can present significant transition problems, especially to the students. In one case, students arrive on campus, presumably with strong manipulative skills, and suddenly they are faced with the expectation that they have to think deeply about and fully understand the mathematics, and that they cannot succeed just by memorizing procedures by rote. In the other case, the students arrive on campus expecting to expand on their understanding of mathematical concepts, to apply mathematics to more sophisticated realistic problems, to use technology, and to work collaboratively in teams. When they are faced with courses that focus almost exclusively on skills and the expectation that they need to memorize procedures by rote, the effect is comparable to running into a brick wall.

Unfortunately, in practice, things are not quite this clear cut. Very few institutions can be selective enough to choose students with any single type of mathematical background. Thus, most schools need to think through how to deal with students having all sorts of different mathematics background, but few are doing so.

However, the transition problems involve considerably more than differences between school and college mathematics offerings. Perhaps the most significant, yet often overlooked, aspect of transition is the issue of placement—the interface between the two. What are the usual placement tests that decide how much students know and what courses they are placed into? There are several widely used, standardized placement tests, which are all based on the traditional school curriculum and are designed to assess students' ability at algebraic manipulation. Also, many mathematics departments use home-grown tests, which likewise typically focus on the traditional high school curriculum. All of these placement vehicles are fine for the first scenario listed above, but what of the other three scenarios?

For instance, one of the national placement tests typically starts with a component measuring a student's ability in algebra. Students who do well are automatically moved on to a higher level component that tests college level (precalculus) mathematics; those who do poorly on the algebra level are automatically moved down to a lower level component testing arithmetic ability. The algebra portion of this test covers 12 topics in an adaptive manner:

- 1. Square a binomial.
- 2. Determine a quadratic function arising from a verbal description, e.g., area of a rectangle whose sides are both linear expressions in x.
- 3. Simplify a rational expression.
- 4. Confirm solutions to a quadratic function in factored form.
- 5. Completely factor a polynomial.
- 6. Solve a literal equation for a given unknown.
- 7. Solve a verbal problem involving percent.
- 8. Simplify and combine like radicals.
- 9. Simplify a complex fraction.
- 10. Confirm the solution to two simultaneous linear equations.
- 11. Traditional verbal problem-e.g., age problem.
- 12. Graphs of linear inequalities.

Now picture what happens to students who have come through a *Standards*-based high school curriculum. Such a student has likely developed an appreciation for the power of mathematics based on

understanding the concepts and applying them to realistic situations, as illustrated in some of the lovely examples and problems described in several of the accompanying articles in this volume, such as Dan Teague's or Eric Robinson and John Maceli's. But, this type of traditional placement test clearly ignores much of what they have learned in the way of non-manipulative techniques, of conceptual understanding, and of contextual applications. So, what happens when such students sit down to take a traditional placement test, which is designed only to determine how many manipulative skills the students have retained? Is it surprising that many such students end up being placed into developmental mathematics offerings because their algebraic proficiency is seemingly very weak? This is certainly unfair to students if they were never exposed to some of those skills, or if the emphasis on those particular skills was lower than in the past to make time for more important mathematics or if the students' experience in mathematics has led them to think of mathematics as something considerably more important, more practical, and more intellectually demanding than squaring a binomial. The result is that many students are placed one, two, or even more semesters behind where they likely should be placed based on the amount of mathematics they took in school.

Furthermore, the standardized tests and most of the home-grown tests deny students' use of technology, even though that had been an integral part of their mathematical experience in high school. (Supposedly, some of the national placement tests will soon allow students to use any standard calculator, including most graphing calculators.)

It certainly seems unreasonable to take students who have completed two, three or even four years of high school mathematics and place them into low level developmental courses because their algebra skills are weak. That weakness is perhaps because those skills may not have been emphasized or perhaps because those skills have grown rusty due to a long lay-off since the last math course in high school. All too often, both courses and textbooks assume a blank-slate philosophy, presuming that the students have never seen anything previously. That is not likely the case and will be less the case in future as the reported percentages of students who continue on to successive mathematics courses in high school increases. (Historically, the drop-out rate was on the order of 50% each year; recent evidence indicates, for instance, that the drop-out rate from first year algebra to second year algebra is now on the order of 10–15%. For additional data, please see "High School Overview and the Transition to College," by Zal Usiskin, in this volume.) It seems that a better solution would be for departments to rethink some of the "remedial" courses they offer to see if they are reasonable based on the overall mathematical backgrounds of today's students.

Now picture what can happen with students who took traditional mathematics courses in high school and who are going into reform courses. On the basis of these traditional placement tests, the students' level of manipulative skills may well be assessed as high enough to place them into courses that are well above the level of their conceptual abilities. If they have never had to understand the mathematics they have apparently mastered and have never been expected to read a mathematics textbook, these students may well be overwhelmed by the intellectual expectations of a reform course. (We would not dream of putting a student coming out of elementary algebra into a course in linear algebra; although the student might have the necessary skills, he or she would need to develop a much higher degree of conceptual ability.)

To illustrate just how bizarre these issues can become, consider the situation in New York state. Over 20 years ago, the State Education Department implemented the Sequential Math curriculum, whose *content* is much in the spirit of the NCTM *Standards*. (Effective in 2001, the state began to implement a new version of this program, a pair of courses called Course A and Course B.) However, apparently not a single college in the state has changed its mathematics offerings to reflect what their in-state students are actually taught in the Sequential Math curriculum, nor the nature of the mathematical experiences that the students came through. Moreover, most of the colleges in the state use the standardized, national placement tests that are based on the old syllabus. Some use home-grown tests, but they are typically as traditional in what they seek to assess.

For instance, the author's school and two neighboring institutions all use the same national placement test, which is designed to assess what students learned from a traditional curriculum that has not been offered in New York for over 20 years. So countless students are being declared "remedial-level" and being penalized for not knowing things they were never taught. Moreover, the mathematics curricula at these three neighboring institutions differ markedly. The curriculum at one school is totally traditional, mirroring the old New York state curriculum, so the students are being squeezed through a filter that has little validity for their backgrounds. At a second school, the curriculum is reform from precalculus up, while at the author's school, the entire curriculum is totally reform starting at the development math level. Thus, at the latter two schools, the students not only are being squeezed through a filter that has little validity for their backgrounds, but also they are being squeezed through a filter that has little validity for their backgrounds, but also they are being squeezed through a filter that has little validity for their backgrounds are being squeezed through a filter that has little validity for their backgrounds are being squeezed through a filter that has little validity for their backgrounds, but also they are being squeezed through a filter that has little validity for mathematics courses they are about to take. Our department has been trying to address the placement issue, but has encountered resistance from the placement office, which does not want to implement a new test, and we have been unable to identify a computer-administered test that reflects our philosophy and needs.

To illustrate just how poorly these tests can assess what students have learned in high school, some 15 years ago, when the author was on staff at one of these neighboring two year colleges, the school first adopted and implemented one of the two national placement tests. Just as the fall classes were about to begin that year, the then-department head discovered that more than 140 entering students who had taken some calculus in high school had been placed into developmental arithmetic by this placement test. The test just kept finding the weaknesses in the students' mathematical ability and eventually traced them all the way down to things like problems with manipulating fractions. To avoid this issue subsequently, the department simply re-normalized the results of the placement test. That is, the bar was significantly lowered—the cut-off scores needed for placement into the various courses were lowered sufficiently to assure that appropriate numbers of students would be placed into each course.

Reportedly, the test-makers such as ETS (Educational Testing Service) have been under pressure to develop a new generation of national placement tests that are more aligned to *Standards*-based courses. That would certainly be a huge step in easing the transition problems. However, the process of developing, testing, and validating such tests is a long-term undertaking and we probably cannot expect to see such products available in the immediate future. Unfortunately, departments that depend exclusively on such tests—most likely because of the ease of administering them to large numbers of students—probably can't do much until then.

However, there are some adjustments that can be made rather simply in terms of placement. For instance, some departments have a placement scheme that utilizes the number of years of high school mathematics that a student has taken and his or her ACT or SAT score in conjunction with a placement test to decide on the appropriate course. Other departments take the number of years since the student's last math course into account in placement decisions. In fact, the author is aware of one large scale study conducted some 10 or 15 years ago at a large two year college where about 18 different factors, including placement test score, SAT or ACT score, age, last math course, and years since last math course were all studied in terms of being effective predictors of student performance. They found that about 12 of the factors were statistically significant and so developed a multivariate regression formula for prediction based on all the relevant factors.

There is one other factor that may be particularly relevant today in terms of the new emphases in reform courses. The greater stress on conceptual understanding, on real-world problems, and writing and other communication skills requires a significantly greater level of verbal ability on the part of the students. As such, it is reasonable to link the score on the verbal/English portion of a placement test with the mathematics score. For instance, the author's department has considered ways to add extra points to a student's math score based on high levels of performance on the verbal portion of the placement test. We believe that the verbal ability will likely compensate, to some degree, for relatively low math scores achieved by some students on such a traditional test.

In the meantime, there is much that departments that give their own placement tests can do to help alleviate many of these problems. The first step is to recognize that they will likely need two different placement tests, one for students coming out of a traditional high school program and another for those coming out of a *Standards*-based program. (Alternatively, such departments might try to develop a single placement test that covers both sides and is designed in such a way that the faculty can interpret the results based on their own needs.) The key is to find ways to identify which student is which; it is unlikely that most students will be able to identify the kind of program they went through.

The second issue is to determine the appropriate mix of problems that are mechanical in nature versus those that are conceptual in nature. In a department offering reform courses, just what are the key manipulative skills that are necessary to succeed in those courses? Is it necessary, for instance, to be able to add or divide relatively complicated fractions, say $\frac{5}{24} + \frac{3}{16}$ or $4\frac{3}{4}/1\frac{5}{8}$, or should students be allowed to convert such expressions to decimals and get the answer using a calculator? Is the ability to get the right answer as important as the ability to look at the second expression and estimate that the value is about 3? (Or should the inability to perform such operations relegate students to a course in remedial arithmetic despite their having successfully completed three years of high school mathematics?)

Then there is the reverse issue. What are the key conceptual skills that are necessary to succeed in a reform course, especially for a student coming out of a reform curriculum in high school? How do you determine if a student truly understands the notation, say for a function, or can only move the symbols around mechanically? How do you measure whether a student has the verbal ability to handle the emphasis on mathematical concepts? Perhaps it would be desirable to use the score on a verbal or English placement test in conjunction with the score on a math placement test.

On the other hand, if a department offers only reform courses, how should it assess the skills of a student who has undergone a traditional high school preparation? If a student lacks key conceptual skills because they were not stressed in high school, but has extremely strong manipulative skills, is a reform college algebra or precalculus course the appropriate solution? Must each such course begin with a review of fundamental concepts that students are expected to know?

And, finally, if a department is offering only traditional courses, how should it assess the non-traditional skills that students may bring to the courses? If a student lacks facility with algebraic manipulations because they were not stressed in high school, but has a relatively deep understanding of the mathematics, is a standard remedial course the appropriate solution? Similarly, if a student has a much broader mathematical experience that includes, for instance, an understanding of statistics, data analysis, and probability, does a standard remedial course make sense? If the answer to these questions is "no," how should such courses be redesigned to build on what such students have learned? Can courses be created that emphasize the development of algebraic skills that take advantage of some of the relatively sophisticated knowledge and experience such students bring instead of treating them as individuals who have never mastered any mathematics? Certainly, if such courses can be designed, they would have a much better impact on the students in terms of both motivation and morale.

Clearly, if we can ease the mathematical transitions of the students, we would make things better for all of us. The students will be better served when they arrive on campus; enrollment in "remedial" courses may actually diminish because many of the students being placed there may not really need remediation; enrollment in college-level mathematics offerings might even increase. The students will be happier, the faculty will be happier, and the administrators will be happier.

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Changing Technology Implies Changing Pedagogy

Lawrence C. Moore and David A. Smith

Duke University

Duke University

Introduction

Sam looked up from the stack of orders on his desk and glanced at his watch. 3:30, time to work on his project with Andrew. He pushed the orders to one side and turned to his computer. No picture this time, but Andrew's voice came through with sounds of students playing frisbee in the background.

Sam, 28, was a non-traditional student, fitting his course work around his work schedule. Andrew was a traditional first-year student. The two had been partners now for four weeks—though they had never met in person.

The background rock music ceased, and Sam heard Andrew's voice, "Hey Sam! What's it like in the real world today?"

"Usual thing, pushing paper... Better watch out for wild frisbees," Sam replied.

"Right. But it's too nice to stay inside," Andrew explained.

"You kids have a soft life," Sam teased. "OK, let's get started."

The instructor's discussion of the project flashed up on the screen. It was just text—Professor Rodriguez was not much for adding voice descriptions. Not like Sam's political science prof, who always added a video stream with her verbal instructions.

Sam proposed a plan of work. "OK, we need to find a picture of a cross-section of a chambered nautilus, then construct a model of the shell's spiral curve. And then we compare it with the real thing. Why don't you search the Web for a good picture, while I look through our class notes for the right formulas?"

Andrew agreed, and his end of the connection went dead. Sam entered a search query, refined it, and found what he wanted. He opened a computer algebra worksheet, made some notes, copied in code, modified it, and produced a test graph.

Andrew's voice returned, and a great picture of a shell appeared in the communication window. "I've put in the x- and y-axes. You can see. And here is a table of coordinates that I pulled off the picture with that cursor widget.... Oh great, you are all set with the modeling function. Right, exponential growth. I worked through that lesson last week. Now how do we match that up with the coordinate data?"

After another fifteen minutes of trial and error and a return to the class notes, the graph of the model function fit well—except for a stretch near the center that just wasn't the same as the rest of the spiral growth pattern.

"Let's ask Rodriguez about this center stuff," suggested Andrew. They quickly drafted a question, attached the picture and the worksheet, and e-mailed the lot to their instructor.

"Once we hear back from Rodriguez, we need to write up the report. Should be able to wrap this up in another hour," summarized Andrew.

Sam heard the rock music resume and then the dull thunk of a wayward frisbee catching Andrew in the head as he bent over his wireless notebook. The communication screen vanished just as Sam's boss showed up at his desk with another pile of orders.

Our scenario is only partly fanciful. For over five years we have had students working on a project similar to this—albeit in a classroom environment with help available from the instructor. The team project in the scenario could have come from our Equiangular Spiral module [5] with some minor changes. (For example, we continue the project into its calculus implications, and we supply the picture.) Indeed, if Andrew did a Google image search for "chambered nautilus," he would have found over 200 great pictures, one of which is the one we used. And if he searched for "spiral" at the math.duke site (not a likely choice, to be sure), he would have found about 20 images, one of which is the picture on which we ask students to do their measurements.

Note in particular the following features of the scenario:

- Assigned group work
- Remote collaboration
- Use of the World Wide Web as an integral part of the project
- Traditional and non-traditional students working together in real time
- Time on task outside of classroom hours, but with (asynchronous) contact with the instructor

Students with notebook computers connected to a campus backbone by wireless cards are increasingly common. Extensive use of communication technology such as NetMeeting is less common but should be the norm in a couple of years. And, if this were an interactive, online article, we could provide a live link to video of students working through our module.

If our scenario is an accurate glimpse of the future—and we believe this future is almost upon us—what are the issues for student learning? We will discuss the following issues in this paper:

- 1. Learning and working in an increasingly rich technological environment
- 2. Making sense of mathematical information—using technology to check
- 3. Student-to-student interactions
- 4. Creation of interactive learning materials
- 5. Intellectual demands of these new forms of learning

Learning and working in an increasingly rich technological environment

Technology is changing the way students approach learning. Increasingly, they will conceive of their work in terms of interactive learning materials, computer algebra systems, spreadsheets, and Web-based cooperation—with occasional use of pencil and paper. Learning how to learn in this environment is as important as learning about the mathematics itself.

Of course, technology has changed how we work and think about work in many ways. Let us illustrate with an example. Suppose you are thinking about writing a paper. You have a couple of ideas; possibly you jot them down on a pad. Then you want to expand them, so you make some more notes, circle them, and draw an arrow to the spot where they should be inserted. Reading the change to be inserted, you realize that other sentences need to be changed as well, and so on. Soon you have several sheets covered with words, lines, loops, and arrows that look more like an abstract painting than a draft of a paper. You quickly abandon paper and resort to a word processor to straighten things out. The point is not so much that you eventually used the technological tool, but that right from the beginning you were framing your

thoughts about the paper with the use of the word processor in mind. Technology has changed the way you conceived of the task, as well as the way you carried it out.

Just as technology has changed the way that most of us approach a writing task, it also is changing the way students think about mathematical activities and carry out mathematical investigations. Graphs are now easy to display and can guide an investigation rather than just be an end product of a difficult calculation. With a symbolic calculating system, long trial calculations are also relatively easy and can also serve to guide an investigation. Similarly, data can be gathered, plotted, and compared. Now the important issues become what calculations, graphs, and data to display and how to interpret them.

Making sense of mathematical information-using technology to check

While it is true that technology will enable students to work with their favorite mathematical representations symbolic, graphical, numeric—it is even more true that students will need to learn how to work and think productively, using many different modes of representation. Indeed, learning how to work and think in multiple representational modes may be one of the most important learning goals of mathematics courses in the age of technology.

In the old pencil-and-paper days, each calculation was likely to be long and subject to errors. Checking, if it was done at all, was likely to consist of performing the same calculation over again—probably making the same error. Now, complicated calculations are easy, and, more importantly, many new ways of checking are readily available. One can compare the symbolic derivative with a difference quotient calculation, a symbolic integration with a numeric integration, or a model function with data. Indeed, modeling provides a strong incentive for students to check their work and correct their mistakes. A student who is not bothered by a pencil-and-paper calculation of a negative volume is much more unsettled by a graph of a model function that does not lie anywhere near the data.

Since students have less emotional attachment to a short computer algebra system calculation than to a long pencil-and-paper one, they are more willing to check the result. They are not looking at the possibility that another 15-minute calculation will have to be repeated. With the pain of checking largely mitigated, the teacher is free to make checking a requirement—and to build checking strategies into the content of the course. Think of the consequences: Getting a confirmed right answer every time will be a normal expectation for both teachers and students. That means we will have to abandon the bell-shaped grading curve—which was never a scientifically sound idea anyway. But it also means—if we have the will—we can eliminate high withdrawal/failure rates and turn mathematics into a subject in which students expect to succeed.

The National Research Council study *How People Learn* ([1], [3]) identifies self-monitoring as one of the key findings from research about successful learning. Specifically [3], p. 13, "A 'metacognitive' approach to instruction can help students learn to take control of their own learning by defining learning goals and monitoring their progress in achieving them." The concept of confirming every mathematical calculation is a local implementation of this principle, since most students start most assigned tasks with the goal of getting the right answer.

This self-monitoring function, known to be important for learning in general, takes on added importance with ubiquitous Web access. Using the Web, a student may find many others who have already dealt with the problem under consideration. How can one know which calculations or conclusions to trust? The ability to evaluate information, to decide what is reasonable, what is correct, is vital to making intelligent use of Web resources.

One illustration of this is [8], a page of lecture notes for a mathematically oriented biology course. This page contains a lot of apparently correct and useful information, but it draws an incorrect mathematical conclusion—one that is obvious to a mathematician but that would easily fool a student. Specifically, Sugg analyzes the classical Lotka-Volterra predator-prey model (in its differential form, not a discrete model) and

concludes that the model is inherently unstable. Nowhere on the page is there any hint of the population cycles that are the correct trajectories of the differential system—and also the observations from nature that motivated both Lotka and Volterra.

Another related issue, one that is general across the sciences and engineering, is created by simulations that eliminate the need to perform real physical experiments. This becomes increasingly important in mathematics as modeling becomes a central part of mathematics courses. The issue is not just the accuracy of the simulation, but also the student's conception of the physical world. What models and data are being used to create a given simulation? How reasonable is it that the simulation accurately represents the aspect of the world under consideration? How can one check?

Student-to-student interactions

A particularly important challenge of this new environment will be designing learning experiences that support cooperative work and the development of a class-wide community of learners. One way to go about this is described in [7] in the context of a differential equations course–but the same principles could be expected to work with lower-level courses as well.

As we imagined in our opening scenario, there will be great opportunities for productive cooperative work—even for students with little or no opportunity for face-to-face contact. In addition to Microsoft's *NetMeeting* (http://www.microsoft.com/windows/netmeeting), many other ways to accomplish real-time collaborations are now available. Some other examples include

- Blackboard (http://www.blackboard.com)
- WebCT (http://www.webct.com)
- Netopia's Timbuktu (http://www.netopia.com/en-us/software/products/tb2/)
- AT&T's Virtual Network Computing (http://www.uk.research.att.com/vnc/)
- Interwise's Enterprise Communications Platform (http://www.interwise.com/)

The capabilities of these products are all different from one another, as are their prices, but each enables collaborators to share work in real time via the Internet.

In the other direction, there is a tendency for technology to provide the individual with a personal learning environment, insulated from contact with others. With headphones delivering a stream of background music and individual hand-held computing devices replacing workstations that can accommodate two people, the individual student may retreat from any significant learning interaction. It will be important for both curriculum developers and instructors to focus on this issue.

Creation of interactive learning materials

What are the implications of technology for developers of learning materials? In the recent past, individual faculty have been creating interactive class materials shortly before they were needed in class. Then, more often than not, the materials were left alone until the next time the author-instructor was teaching the same course. Even if an author did more work, it was unlikely that the materials were ever "finished" in any reasonable sense. In some ways, this is comparable to the period in the 1970s when many individuals wrote their own word processing programs. After a short transition period, users came to expect more from a word processing program than most individuals were willing or able to produce. Now most of us use one of the common commercial programs.

For learning materials, there are currently two trends. One is for teams of individuals to work together to produce materials that include sophisticated interactions delivered in a setting that is easy to use and very flexible. The other trend is similar to the phenomenon of open-source software. Authors cooperate in a loose federation that combines compatible learning components in different ways as necessary and leaves the product for further development by others.

In the old textbook-oriented model, a small group of authors, working very intensely, produced most of the major text material. The individual faculty member's responsibility was to create a syllabus around the published text. Now, regardless of the interactive materials used, the instructor is going to be much more closely involved, often adapting the materials for his or her own use. Beyond that, many more instructors will be part of the design and development of the materials. However, if it is done well, the development of learning materials that incorporate technology will take extensive time and effort. How are authors to be rewarded? The rewards will probably not be royalty income so much as scholarly recognition. So far, this sort of recognition has been slow to develop.

Intellectual demands of these new forms of learning

Finally, we need to be clear that students will be expected to do more challenging tasks than in the past—particularly in precalculus and calculus courses. In the past, just deciding on a symbolic calculation algorithm and executing it with care represented a satisfactory learned response. Now the student will need to recall and evaluate the usefulness of and connections among a variety of representations and computations. This is a higher-order intellectual activity—one that will allow learning at a deeper level. Fortunately, reforms such as the NCTM *Principles and Standards* [6] have paved the way for this change.

It is no longer acceptable to assess student learning by asking them to solve calculational problems because computer algebra system (CAS) capabilities are widely available to almost everyone. For example, the Texas Instruments TI-89 (about \$150) provides powerful algebra and calculus capabilities (with 2-D and 3-D graphics) in a handheld calculator. Many schools and colleges provide site-license access to *Maple*[®] or *Mathematica*[®]. *StudyWorks* (essentially a fully functional version of *Mathcad*[®]) is available from Mathsoft for about \$40. And there are a number of free or inexpensive online services that will accept a problem input and provide the output from, say, *Maple*[®] or *Mathematica*[®]. One example is *The MathServ Calculus Toolkit* at Vanderbilt University [2], which includes a number of precalculus topics as well. Simply forbidding the use of any of these tools is about as effective as sticking a finger in a crumbling dike.

In fact, it never did make sense to assess student understanding of mathematics solely or primarily by their ability to do unaided symbolic calculations. At best this ability is a poor proxy for understanding, as anyone can learn simply by asking students to explain what they are doing as they carry out a calculation. And generations of students have come to believe that the calculations are what mathematics *is*. Worse, reserving the rewards for those who are proficient at calculations in a timed, closed-book, no-technology test setting has denied success to many other students who are quite capable of understanding mathematical concepts—as we have learned by teaching those students in technology-rich environments. Whatever the limitations on our profession in the past, we are not condemned to repeat failing practices forever.

On the positive side, a recent analysis and synthesis [4] of research on the use of technology in mathematics instruction at all levels has documented strong support for welcoming technology as a component of our pedagogical practices. One of us co-authored the calculus chapter [9] in this volume, which includes among its conclusions the following:

- "Technology integrated intelligently with curriculum and pedagogy produces measurable learning gains...."
- "There is evidence that using tools such as Mathematica and Maple for conceptual exploration ... leads to conceptual gains in solving problems that can transfer to later courses. In comparison, students following traditional courses tend to use more procedural solution processes."
- "Technology enables some types of learning activities (e.g., discovery learning) and facilitates some others (e.g., cooperative learning) that are harder or impossible to achieve without technology."

Of course, the completed research all refers to technologies that have been available in the past. The technologies becoming available to us now hold promise for even more exciting gains—if we can keep up with the intellectual challenge of adapting our pedagogies to the realities of the world in which our students live.

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25

Preparing for Calculus and Beyond: Some Curriculum Design Issues

Al Cuoco

Center for Mathematics Education–EDC

This paper outlines an alternative to the topic-driven design principle that is the basis for most precalculus courses, arguing that the real power of mathematics lies in the methods used to produce results as much as in the results themselves. It describes a fourth-year high school course that adopts this design, with examples and student work.

Introduction

Curriculum design in US precollege mathematics is largely topic driven; a course is defined by the topics it treats. The major criteria for including a topic in any particular course include:

- does it review and deepen important ideas from previous courses?
- is it a prerequisite for likely subsequent courses?
- did it fall through the cracks in earlier grades?
- does it appear on high stakes tests?

As one moves up the grades, the effects of this design principle compound. By the time one reaches the fourth year of high school, we end up with 18-chapter, 800-page compendia of topics that range from trigonometry to data analysis to complex numbers. These monster texts all go under the name "precalculus," which is therefore defined as everything from trigonometry to data analysis to complex numbers.

Of course, there's much more in these texts than what one needs as preparation for any of the current calculus offerings. Indeed, it's a well-known fact among high school teachers that one can only finish slightly more than half of these chapters in a given year, and yet many students who go on to calculus from such experiences have what they need to get respectable grades.

But in addition to being too big, these courses are, at a deeper level, too small. There has been a growing consensus among all involved in secondary mathematics education that this topic-driven curriculum is not serving our students well. Today's high school graduates enter a highly technological world in which mathematics plays an essential role. However, the widespread usefulness and effectiveness of mathematics in fields outside the discipline come not just from mastering specific skills, topics, and techniques, but more importantly, from developing the ways of thinking used by scientists, mathematicians, engineers, and others. National reports and standards documents ([1], [2], [3], [4], for example) have articulated a sentiment that is widely held by people in the mathematics education community: essential student outcomes from a modern mathematics curriculum should include skills like:

- finding and analyzing patterns,
- designing and conducting experiments,
- describing and communicating,
- tinkering and inventing,
- visualizing and conducting thought experiments,
- conjecturing and guessing,
- theorizing and abstracting, and
- making logical connections and explanations.

See ([6], [7]) for an elaboration on this list. Over a decade ago, *Everybody Counts* [5] described it this way:

Mathematics offers distinctive modes of thought which are both versatile and powerful, including modeling, abstraction, optimization, logical analysis, inference from data, and use of symbols. Experience with mathematical modes of thought builds mathematical power—a capacity of mind of increasing value in this technological age that enables one to read critically, to identify fallacies, to detect bias, to assess risk, and to suggest alternatives [p. 31].

Some very useful "modes of thought" in mathematics are given short shrift in high school (and especially in precalculus courses): hardly showing up at all are reasoning about algorithms, combinatorial thinking, and using the linearity of certain maps on the plane. Furthermore, even for students who go on to calculus and advanced mathematics, the emphasis on traditional precalculus skills and methods is misplaced. Calculus instructors have long complained that the real stumbling blocks for their students are the hard *ideas* in the subject: notions like limit, approximation, convergence, and error estimation.

Organizing curricula around these *mathematical habits of mind* provides an alternative to topic-driven design. It provides another criterion for including or excluding a particular topic, and, what's more important, it has a great influence on how topics are developed: Explicit attention is given to the methods behind the results.

In mid-1996, NSF convened a group of curriculum developers, mathematicians, and educators to discuss possible directions for fourth-year high-school mathematics. At the time, NSF had invested heavily in comprehensive curricula for the first three years of high school. It was time to look at what belonged in the bridges between high school, postsecondary programs, and the workplace. At that meeting, participants expressed a need for problem-based and student-centered materials that:

- build on and make use of the rather different backgrounds that students would develop in any of the three-year "standards based" programs,
- identify and formalize "big mathematical ideas" developed during the first three years of high school (for example, a general notion of function or proportional reasoning),
- use mathematics itself as a context for developing mathematical ideas, and
- treat topics and thinking processes that are prerequisites for postsecondary education and for schoolto-work transitions (trigonometry or algorithmic thinking, for example) in the spirit of reform without losing their essentially technical nature.

It was clear at this meeting that there were many different topics, organizing principles, and viable directions that could be taken in such a curriculum for the latter part of high school. Everyone agreed that the last years of high school, more than any other part of the K–12 curriculum, called for a multiplicity of approaches and options, not only for the preparation of future scientists and engineers, but also for developing informed citizens.

My colleagues and I at EDC's Center for Mathematics Education decided to apply our design principle putting mathematical thinking at the center of curriculum development—to the creation of a fourth year course for high school seniors. More precisely, we wanted to develop a program that would:

- give students a sense of what mathematics is about and the experience of what it is like to do mathematics,
- help students develop the habits of mind used to create the major results of modern mathematics, and
- prepare students for future work in mathematics and science, should they choose to pursue those fields.

This paper describes that attempt, an attempt that evolved over the course of several years by adding constraints imposed by the field, making compromises imposed by the nature of schools, and incorporating the brutal and enlightening feedback one gets from field-test teachers and (especially) students. The result is a course for high school juniors and seniors, appropriately named *Mathematical Methods* (or M², for short) [11], that looks quite different from traditional precalculus courses, in spite of all the compromises we made.

Goals for the program

While mathematical thinking was our primary goal, there were several others at play. Some are deeply rooted in our approach to mathematics education, while others are more focused on the particular audiences for this course. Like all projects just starting out, we spent some time creating a list of lofty (and rather poorly defined) goals for the work:

- 1. The materials should center around mathematical thinking.
- 2. The materials should be accessible to our intended student and teacher audiences.
- 3. We should set high expectations and help people meet them.
- 4. Each chapter should be easy to start and should take students farther than they dreamed possible.
- 5. The chapters should form a web of interconnected ideas.
- 6. We should use the most effective technology available for helping students develop the habits that we want to foster.
- 7. The materials should help students see the value and importance of mathematics.
- 8. We should work to enlarge students' and teachers' notions of reality to include mathematics.
- 9. Graduates of our course should love mathematics.
- 10. The benchmarks for choosing a topic include:
 - it provides an opportunity to develop mathematical ways of thinking
 - it is of historical importance
 - it is useful
 - it is beautiful
 - it prepares students for future mathematics
 - it contributes to a broad picture of mathematics
- 11. The materials should be useful across the wide range of upper high school courses.
- 12. The materials should be faithful to mathematics as a discipline, emphasizing major historical themes, results, and problems.

Looking back, these seem grandiose, subjective, and a little naive. Some of them seem to pull in opposite directions. Seven years later, there's a story to tell about what happened with each of them. We'll tell a few of these stories in the last section of this paper.

What habits? What topics? What students?

There's an old saying that you can't think about thinking unless you think about thinking about *something*. A course organized around mathematical thinking would be little more than a mathematics appreciation course if it contained chapters on "doing experiments" or "using linearity." Our intention was to use mathematical topics as vehicles for certain ways of thinking. But, of course, the topics needed to serve other purposes as well—the ones laid out in item 10 above.

And what was the audience for the course? As other papers in the proceedings have documented so well, there is wild variation in the reasons students take a fourth year of mathematics. At one extreme are the students who aren't interested in (or lack the grades for) a college-prep precalculus or calculus course. These students typically take courses, with titles like Senior Topics, that are just watered down versions of precalculus, with some SAT review thrown in. At the other end are students who've taken all the AP the school has to offer and want an advanced elective. These students often find haven in a computer science course, a directed study with a willing teacher, or a college level course at a local college. In the middle are most of the college-bound students, taking precalculus, calculus, or one of the AP calculus courses.

The design therefore turned into a three-dimensional effort:

- What students will we serve?
- What mathematical habits do we want students to develop?
- What topics are good vehicles for developing these habits and serving the needs of students after they finish M²?

Based on the belief that the vast majority of students are capable of serious mathematics, we decided to develop materials that could be used across a wide spectrum of fourth year courses, from the topics courses to the advanced electives. This led to some early decisions:

- M² would be designed for students who have either completed one of the comprehensive high school curricula or a more traditional Algebra 2 course.
- We would aim the materials at:
 - secondary students who intend to work in a technical field;
 - college-intending students who are looking for an alternative fourth year of mathematics;
 - students who wish to take a mathematical elective during their junior or senior year (either instead
 of or in addition to precalculus/calculus sequence).

We also wanted to address the needs of students who may not be planning to continue their mathematical studies.

To reach such a broad audience we borrowed a structure from an earlier curriculum effort [14]: We would write a small number of large chapters around big themes. Each chapter would start with an extremely simple entry point that would be tractable for almost every student and then would carry the development to levels to challenge the most advanced students. The idea was that each chapter would have certain jumping off points, so that teachers could customize the materials for their students in a way that, instead of forgetting the last half of the book, allowed them to help all students to experience the important themes, develop the central mathematical habits, and dig into the major results in the course. This decision was by no means unanimous. As one staff member put it, "It sounds to me like we're willing to take on students of widely different backgrounds and prepare them for very different future mathematical careers. I'm not sure just how feasible this really is." None of us was sure, but we decided to see how far we could take this approach.

As for mathematical ways of thinking that we wanted to foster, we decided on "the big four":

- algebraic thinking
 - designing and using algorithms

- reasoning about numerical and algebraic calculations
- linearity
 - reasoning by linearity (in the sense of linear algebra)
 - using linear approximations
- combinatorial methods
 - recognizing isomorphic combinatorial problems
 - counting without explicit enumerations
- analytic thinking
 - reasoning by continuity
 - making successive approximations

Our benchmarks for choosing a topic, our choice of habits of mind, and this "no threshold, no ceiling" decision put some strong constraints on our choices for chapters. For example, not every mathematical topic has easy entry, and school mathematics is notorious for introducing topics that don't go very far. Our first cut contained four chapters:

- Solving Equations (including polynomial and trigonometric equations)
- Coordinates and Linear Methods (including the use of matrices)
- Counting/Recursion/Sequences and Series (including limits and infinite series)
- The Complex Numbers (building on and using everything that comes before)

Lists are seductive, and we had created several. I was convinced that we had a simple plan for a lean course that would give students a real taste of what mathematics is all about—the rest of the work ahead seemed straightforward. Then we took these ideas out for a spin, in classrooms, with a local advisory board of teachers, and with a national advisory board of high school and college teachers, mathematicians, and mathematics educators.

The Evolution

After consulting with our advisors, drafting sample activities, and piloting the drafts in local schools, it became apparent that the design had to be refined. There were missing topics (that *had* to be there, of course), there were inter-chapter dependencies that couldn't be resolved, teachers were worried that the chapters were *too* big (covering too much material), and the easy entry and useful benchmarks seemed to be violated in some of the drafts—we needed some better hooks. The outline passed through several iterations. We stayed faithful to the habits we wanted to develop and the audience we wanted to reach, but the outline at each iteration began to look a little more like a precalculus course, with more and shorter chapters.

And there was a problem with technology. The writers were convinced that, in addition to a graphing calculator, a computer algebra system was essential for some of the topics, methods, and ideas we wanted to develop. The CAS of choice was one installed on a computer, something like Mathematica or Maple. It was especially important that the system we use be *extensible*, so that students could build computational models of their own for algebraic objects (sequences of polynomials, for example) and so that they could express in the CAS abstract relationships (of *their* invention) between mathematical structures. However, it became clear that M² would not be used, except by the wealthiest districts, if we insisted on a computer-based CAS. Indeed, actual use of desktop or laptop computers in mathematics classes is on the decline among the teachers in our advisory group. Instead, teachers are using calculators. The CAS system on

calculators lacked some of the functionality and power that we thought we needed. I was especially worried about the interface, memory limitations, and notational conventions built into these hand-held machines, seeing many places where students could develop *bad* habits of mind through their use. But it was clear that we needed to adapt to the constraints or give up hope of using a CAS. It turned out that the system on the TI-89 and 92 allowed us to do most of what we wanted (especially with the wonderful support we got from the TI development staff), was extensible enough to get across the idea that one could build computational models for mathematical objects, and had an interface that posed very little difficulty for students. And in at least a few cases, the memory limitations of the machine turned out to be a perfect pedagogical device for developing important methods like mathematical induction (see [9] for details).

After several iterations, we ended up with a book containing seven chapters—more than we wanted but less than the norm. We had to make some hard decisions about what to leave out. The linearity of rotations and summations remains, but a more general discussion of linear transformations fell out. We didn't get as far into difference equations as I had wanted. I had hoped that we'd take a much more structural approach to complex numbers. And we just touched on ideas that I wish could have been given better play: Applications of complex numbers to the theory of regular polygons, and the mathematics behind public key cryptography. Many of these decisions to leave topics out would have been made differently had we been using topic-driven design, but the key focus on mathematical habit became the arbiter that caused many beautiful topics to be put aside. Other decisions came about from purely practical issues. For example, in the last months of the development, we did a major rearrangement of the *order* of the chapters, because teachers said that, especially in senior classes, technical material (like trigonometry) should never come near the end of the year. And, of course, we had to make sure that students would be prepared for calculus.

Here's a brief description of the seven chapters:

(1) Tables, Patterns, and Rules. This chapter asks students to find functions that agree with input/output tables. Students generate closed form and recursive rules for these functions, and then learn about mathematical induction as a way of showing that two seemingly different functions agree on the non-negative integers (see [10] for an elaboration of the approach). Advanced topics include methods for finding genuinely *different* functions that agree on a table and Lagrange interpolation. The focus is on algorithmic thinking, finding and describing patterns, and algebraic thinking.

(2) Polynomials. This is a chapter on advanced algebra. There's a dual focus: Polynomials as formal objects (algebraic thinking and reasoning about calculations) and polynomials as representations for certain continuous functions (reasoning by continuity and analytic thinking).

(3) Complex Numbers and Trigonometry. The typical way to establish the geometric interpretation of complex numbers is to use the addition formulas for sine and cosine. This chapter goes the other way: By establishing the linearity of rotations in the plane, one can use the geometry of complex multiplication to derive the addition formulas for sine and cosine (as well as most other trigonometric identities). This has several advantages:

- It shows the essential connection between geometry and the algebra of complex numbers without the heavy machinery of trigonometry.
- It provides an *application* of the algebra of complex numbers: Students develop a general purpose machine for proving trigonometric identities.
- It provides some coherence to the topic of identities, showing how most depend on the invariance of the unit circle under rotation about the origin and reflection in a diameter.

(4) Count It Up. This is a chapter about combinatorics and combinatorial thinking(predicting the outcome of an enumeration without having to carry it out explicitly). Combinatorial formulas are developed in

special cases by solving problems and are then generalized. After developing results about permutations, combinations, and binomial coefficients, the chapter culminates with one large problem—the "Simplex Lock" problem—described at: http://www2.edc.org/makingmath/mathprojects/simplex.asp

(5) Add It Up. The idea here is to build on Chapter 1, looking at techniques for summing series. This can all be done in an informal way, using the kind of mathematical induction developed earlier. Topics include:

- arithmetic sequences and series,
- geometric sequences and series,
- sums of squares and cubes, and
- telescoping sums and how to build them.

Students use their CAS to investigate and prove properties of the \sum operator, many of which have parallels in calculus with \int . This chapter is about algebra and algorithms, reasoning about and transforming calculations.

(6) The Ideas of Calculus. This chapter tells the story of how mathematics evolved to solve the problem of finding areas enclosed by curves. Another message is that functions can't always be given by algebraic formulas. Using techniques for summing powers from Chapter 5, students can tackle the problem of finding areas bound by graphs of polynomial functions. We also develop Fermat's brilliant method for finding the area under *every* curve of the form $y = x^n$ ($n \neq 1$) using geometric series. Again, techniques from Chapter 5 allow students to recreate Fermat's solutions.

(7) Algebra and Cryptography. The chapter starts by looking at some simple and historically important encryption schemes. For example, students look at linear functions $x \rightarrow ax + b$, applied to a letter's position in the alphabet. This introduces the structure of Z/26Z (because if *a* has a common factor with 26, you make a bad cipher). More general ciphers are defined by 1–1 functions on Z/26Z. From here, we will investigate some elementary number theory, including:

- solving equations in modular systems,
- units and zero divisors, and
- Fermat's Little Theorem.

The chapter culminates in a treatment of public key cryptography.

In the next section, I'll describe a student activity that illustrates more precisely how the design principles play out across the chapters.

An example

The course opens with students trying to find rules that agree with tables. As many people who've tried this know, students naturally gravitate to recursively defined rules. So, asked to find a function that agrees with this table:

Input	Output
0	3
1	5
2	7
3	9
4	11
5	13
6	15

students notice both the closed form rule $n \rightarrow 2n + 3$ as well as the recursively defined rule

$$f(n) = \begin{cases} 3 & \text{if } n = 0, \\ f(n-1) + 2 & \text{if } n > 0. \end{cases}$$

While many students notice the recursive pattern, the transition between a verbal description and the above mathematical notation is not easy. Modeling the recursive rule in a CAS helps many students make the transition. In the TI-89 system, the model looks very much like standard mathematical language:

The interplay between recursive and closed form models for functions is a theme that runs throughout the course. For example, early in the year, with no formal machinery behind them, students are presented with the following problem:

Suppose you want to buy a car. You don't have much money, but you can put \$1,000 down and pay \$250 per month. The interest rate is 5%, and the dealer wants the loan paid off in three years. What price car can you buy?

This leads to the question, "How does a bank figure out the monthly payment on a loan?"

A recursive approach lets students experiment using their CAS. They begin with a simpler model: one in which there is no interest. Suppose the original price of the car is 10,000. If b(n) is the balance owed at the end of n months, the model looks like this:

$$b(n) = \begin{cases} \$9,000 & \text{if } n = 0, \\ b(n-1) - \$250 & \text{if } n > 0. \end{cases}$$

Students gradually refine the model to include interest. They translate the following verbal description:

What you owe at the end of the month is what you owed at the start of the month, plus 1/12 of the yearly interest on that amount, minus your monthly payment.

into a mathematical function:

$$b(n) = \begin{cases} 9,000 & \text{if } n = 0, \\ b(n-1) + \frac{.05}{12}b(n-1) - 250 & \text{if } n > 0. \end{cases}$$

Students can then experiment, adjusting the "250" until they make b(36) = 0. In fact, many students add another input to their balance function, so they can change the monthly payment on the fly. A TI-89 model looks like this:

```
: b(n,m)
: Func
: if n = 0 then
: return 9000
```

```
: else
: return (1 + 1/12*.05)* b(n-1,m) - m
: endif
: endFunc
```

Students love to get the monthly payment down to the penny. In fact, we go up one level of abstraction and ask them to calculate monthly payments on several loans, seeing how the payment changes with the cost of the car:

Pick an interest rate and keep it constant. Suppose you want to pay off a car in 36 months. Investigate how the monthly payment changes with the cost of the car.

(1) Make a table like this:

Cost of car (in thousands of dollars)	Monthly payment
10	
11	
12	
13	
14	
15	
16	
17	
18	

(2) Describe a pattern in the table. Use this pattern to find either a closed form or a recursive rule that lets you calculate the monthly payment in terms of the cost of the car in thousands of dollars. Model your function with your CAS and use the model to find the monthly payment on a \$26,000 car. Check your result with the original approximation method.

Each entry in the table is calculated by a series of approximations, and then the entire table is treated as a new data set in which students find a surprising relationship. A snapshot of one student's work can be found on the next two pages.

Michelle notices a linear relationship between the cost of the car and the monthly payment. She doesn't yet have the background to prove that the observed relationship is, in fact, linear, but she has evidence for a conjecture. Indeed, Eric Karnowski, a colleague at EDC, was working through this problem and found an extremely beautiful way to use the full features of the CAS on the TI-89 to get an explicit formula for the monthly payment with no need for successive approximation. The balance at the end of 36 months with a monthly payment of \$250 can be obtained by entering b(36, 250) in the calculator (we get \$764.92 as a balance). Eric thought of m (the monthly payment) as a variable, and he wanted to find the value of m that makes b(36, m) output 0. So, he asked the CAS to simplify b(36, m). Rather than assigning m a value, he left it as an indeterminant and entered b(36,m) in the TI-89. The calculator outputs an expression in m—it gives

10453.250082011 - 38.75333552005m.

But we want this expression to be 0. So, we enter

solve(10453.250082011 - 38.75333552005*m = 0,m)

A 0 a) 730 100 29.7 1 730 2 69) 730 3 8.9.7 >30 1:19.7 4 >30 5 149.7 >30 G 179. >30 7 209.7 >30 8 239.7 >30 9 269.7 >30 299.7 10 >30 11 329,7 730 12 359.7 > 36 389.7 > 30 14 419,7 730 15 449.7 730 16 750 509.7 17 18 539.7 b) $\{-3, 3, F, x = 0\}$ $\{y(x)^{2}, (y(x-1)+30, F, x>0)$ $y(x) = -\frac{1}{30}$ a) \$26000 car \$1779.7 monthly payment -C) y(x) PX=0 Then Return -. 3 Else Return y(x-1)+30 Enafune

• I changed the amount of the Cost of the Car then I changed the monthly payment until I found the right monthly payment. • I found that each time the Cost of the car went up \$1000 the monthly payment went up \$30. yc 5917 Month 534.7 4777 Payment 419.7 351.7 2997 23977 174.7 19.7 71.7 20 56799 lo Cast of Car (thousand

Figure 2.

and find that

$$m = 269.738...$$

A monthly payment of \$269.74 will do the trick. For more examples like this, see [8].

Much later in the year, when studying series, students develop the technique of unstacking a recursive definition to express it as a summation. They apply this technique to the problem of finding an explicit formula for monthly payments. The development goes something like this:

Suppose you borrow \$12,000 at 5% interest. Then you are experimenting with this function:

$$b(n,m) = \begin{cases} 12000 & \text{if } n = 0, \\ (1 + \frac{.05}{12}) \cdot b(n-1,m) - m & \text{if } n > 0. \end{cases}$$

Notice that:

$$1 + \frac{.05}{12} = \frac{12.05}{12}.$$

Call this number q. So, the function now looks like:

$$b(n,m) = \begin{cases} 12000 & \text{if } n = 0, \\ q \cdot b(n-1,m) - m & \text{if } n > 0 \end{cases}$$

where q is a constant.

Then at the end of n months, you could unstack the calculation as follows:

$$b(n,m) = q \cdot b(n-1,m) - m$$

= $q (q \cdot b(n-2,m) - m) - m = q^2 \cdot b(n-2,m) - qm - m$
= $q^2 (q \cdot b(n-3,m) - m) - qm - m$
= $q^3 \cdot b(n-3,m) - q^2m - qm - m$
:
= $q^n \cdot b(0,m) - q^{n-1}m - q^{n-2}m - \dots - q^2m - qm - m$
= $12000 \cdot q^n - m(q^{n-1} + q^{n-2} + \dots + q^2 + q + 1).$

The last series is geometric; summing it, we get

$$b(n,m) = 1200 q^{n} - m \frac{q^{n} - 1}{q - 1}.$$

Setting b(n, m) equal to 0 gives an explicit relationship between m and the cost of the car that explains the conjectured linearity that was noticed months earlier.

This description of the development is a much compressed version of the informal and gradual development that students in M^2 experience. And, in fact, the last part of the development wasn't attempted in several of the field test classes.

The monthly payment activity is typical of the kinds of activities we sought in developing M^2 : Activities with easy entry, fundamental use of technology, and opportunities to develop and prove conjectures, make connections, and experience some basic mathematical methods.

Lessons learned

Developing M^2 has been a learning experience for all of us on the staff. Here are some reflections now that the development is finished.

- Reviews of early drafts of the chapters often contained comments like "high school kids could never do this kind of thing." Field tests showed otherwise. The materials had to be revised, and in many cases completely reworked, but in no case did we need to water down the level of mathematics for either students or teachers. I'm convinced that traditional curricula expect far too little from teachers and students.
- A colleague and I taught sections of the field test at two different local high schools. Observing classes or delivering occasional lessons is important, but the experience of teaching the course every day, in an authentic school setting, seeing first hand both students' fundamental difficulties and breathtaking insights, informs the development process in a way that nothing else can. Three-hour arguments at staff meetings about the sequencing of problems or the potential engagement of students are settled in 10 minutes in a classroom.
- High school students take delight in their own mathematical thinking. In addition to a teacher advisory board, we convened a student board to help us with questions of readability and level. At the first meeting, we asked students what they liked most about M². Now, the students had at this point been fitting functions to tables, proving things by mathematical induction, and working on other activities that are usually classified as pure mathematics. Four of the students immediately responded to our question, almost in unison, with something like "It's realistic." When prompted to explain, it became clear that they liked doing *real work*—work that allowed them to think for themselves and to exercise their creativity. Realistic contexts were nowhere near as important to them as a realistic style of work.
- Along these lines, students want to see how mathematics is used, but applications are just as engaging from inside as they are from outside mathematics. Indeed, by not separating or distinguishing these types of applications from each other, students enlarge their definitions of "real world" to include mathematical contexts, and in the end, this makes them much more able to apply mathematical thinking to all kinds of situations.
- Students at all levels can do this kind of work. Much of the field test of M² was done in senior topics classes; my students were the weakest students in their school taking a fourth year of mathematics. Poor performance in mathematics courses has many causes, but lack of ability to think in a characteristically mathematical way is, for the vast majority of students, not one of them.

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Alternatives to the One-Size-Fits-All Precalculus/College Algebra Course

Bonnie Gold

Monmouth University

Introduction

How do we want our future legislators, our future news reporters, our country's future parents to feel about mathematics? Do we want them to believe it is a collection of rituals, requiring special skills only achievable by a few and of no practical value? Or would we prefer that they see mathematics as a way of describing many aspects of the world, central to many issues that will affect their lives, and a subject in which they can achieve whatever level of proficiency they need?

If the last mathematics course students take is a traditional college algebra or precalculus course, the vast majority of the students leave feeling defeated in their ability to do mathematics and mystified as to the use or value of the subject. As someone at the conference, Rethinking the Preparation for Calculus, said, "There's something very wrong if the last course a student takes in a subject is named 'pre' anything." And yet, there is a wealth of mathematics, accessible to students at this level, that is being applied to a wide range of contemporary issues.

Can one size fit all?

A cornerstone of the American democracy is that all children should be given equal opportunity. Unlike many countries that start directing children to different academic tracks by age 12 or earlier, American education treats all children as potential national leaders. We try to give them a mathematical background that allows them to become top scientists. As a result, the standard school mathematics track leads to calculus. While this may be a reasonable policy at the school level, by the time students arrive at college, they have become unequal in many ways. Some have been stimulated by their school mathematics, while others have been crippled by their early mathematical experiences. Some have a clear interest in a mathematically-intensive discipline, while others are clearly focused on the humanities, business or social sciences and others are still undecided. One size no longer fits all (if it ever did), in college mathematics courses.

At the conference, many felt that the problem is that precalculus (or college algebra) is trying to do too many things for too many audiences. It's trying to prepare students to major in mathematics and the sciences, to be the terminal general education mathematics course, and everything in between, and is doing none of this well. The solution I developed at Monmouth was to break up our college algebra course into several separate courses each with a clear mission and clientele. At institutions where the standard course taken by everyone is precalculus, one could do the same at that level.

When I came to Monmouth University in 1998, college algebra was our largest course. It was taken as the prerequisite for precalculus for students who didn't do well on the placement examination but intended to go on to calculus, as the prerequisite for the business mathematics sequence (a semester of combined linear programming and precalculus, followed by a semester of applied calculus), and was required of biology and social science majors who would continue on to a statistics course. Since all these disciplines required college algebra, advisors used it as the default placement for almost all students. Even students who entered planning on a major not requiring college algebra would thus not have to take a second mathematics course should they change their major. Only students who were absolutely decided on a major in the humanities were placed in the course (Quantitative Reasoning and Problem Solving) the department had developed for general education purposes. By the time I arrived, even our future elementary teachers were often taking college algebra as their only college mathematics course.

In our college algebra course, we try to give students all the algebra they need for calculus. So the course includes linear, quadratic, polynomial, rational, exponential, and logarithmic functions, solutions of equations involving all of these kinds of functions, simplifying expressions involving these functions, and so on. For students continuing on to precalculus (which adds trigonometric functions and does more with functions in general—graphing, inverses, transformations, and so on), all of this material is necessary. We use all of this material in our standard calculus sequence. But students in biology don't need to know how to solve $\sqrt{2x-3} = 5 - \sqrt{x+7}$. They do need to understand linear and exponential growth, and be able to recognize the distinction between them. They must understand the idea of a rate of change, have a bit of appreciation of effects of scaling, and know enough trigonometry to handle vectors in physics. Students in the social sciences need to be able to correctly use formulas in statistics involving sigmas, have an understanding of rates of change, including what units are involved in a particular context, and be able to use and interpret graphs and tabular data. Our business mathematics sequence uses linear, quadratic, exponential and logarithmic functions, but with much less symbolic manipulation than the standard calculus course. Because of the amount of time devoted to algebraic manipulation in the college algebra course, the students in these disciplines were not learning the things they actually needed for their majors and, since they mostly got Cs and Ds on the second or third try, they were not learning symbolic manipulation either.

An alternative: Splitting the course

The best way to break up college algebra or precalculus to serve client disciplines depends on the particular institution's programs and student body. At Monmouth, the college algebra audience was sent in four directions.

- For elementary education majors (who NEVER should have been in the course) we developed a new course, Foundations of Elementary Mathematics that, as the Mathematical Education of Teachers document (Chapter 1 in [1]) strongly recommends, gives these students a deep understanding of the mathematics they will be teaching.
- For biology majors we developed two courses, Introduction to Mathematical Modeling in the Biological Sciences and Calculus for Biologists, only the first of which is required. We hope the better students in the first course will be inspired to continue to the second (and some do).
- For social science majors, we developed Mathematical Modeling in the Social Sciences, which also is an acceptable prerequisite for the business math sequence.
- For students who eventually go on to a standard calculus course, we retained College Algebra.

In addition, we retained Quantitative Reasoning and Problem Solving as our general education quantitative literacy course for students in majors without a specific mathematics requirement. The two modeling courses are at the college algebra level. We study primarily linear and exponential models, with some time spent on quadratic models in Mathematical Modeling in the Social Sciences and on power function models in Mathematical Modeling in the Biological Sciences. These are topics from the college algebra course. However, we replace time spent learning symbolic manipulation by time spent looking at a range of applications related to the disciplines whose majors take the course, including how to interpret answers in terms of the original problem. We use many data-driven projects taken from the media or texts in their fields. We also make extensive use of the computer; each course involves at least seven computer lab projects done by students working in pairs. (This would work equally well with graphing calculators.) We use Excel as the computational tool, since it, or a similar spreadsheet, will be available to most of these students once they graduate and start working. (In addition, all students at Monmouth are required to take an introduction to information technology, which includes some work in Excel.) We assign for homework primarily the problems from the texts that are relevant to students' fields and use examples from those fields on exams as well as in labs.

Before developing these courses, I spoke with the chair of the biology department and then to the whole department at one of its meetings. I also spoke to the chairs of the social science departments, whose students had been taking college algebra as a prerequisite for statistics, and to the faculty member responsible for their statistics course, who is in the psychology department. I wanted to find out, from the perspective of the faculty in these fields, what they needed students to get from college algebra.

For the social sciences, the main skill they wanted was correct understanding of order of operations. They also felt that students in our general education offering didn't develop the necessary level mathematical sophistication. (The students in our quantitative reasoning course are, on average, even more math phobic than our social science majors. Since this is their last mathematics course, and the purpose is, in part, to make them less math phobic, we cover topics rather gently, which doesn't get them to the level of sophistication the statisticians in the social science departments wanted.) The biologists wanted a bit more—their students need some knowledge of exponential and logarithmic functions for chemistry and a bit of trigonometry for physics. I looked at what was available among current textbooks and designed a course for social sciences based on the Kime-Clark text [2], and one for biology based on the Crauder-Evans-Noell text [3]. Once I had a tentative syllabus and text, I again discussed the courses with the client departments to ensure the new courses would meet their needs. The departments were delighted to be consulted by the mathematics department about what they wanted for their students. They were very cooperative and helpful and promptly changed their requirements to make these courses required of their majors.

Students are much more responsive in these courses than they were in college algebra. They can see the use of the mathematics they are learning and are not overwhelmed by attempts to learn too many techniques in too short a time. There are some problems that don't go away. Most students take these courses in their first semester or two, and many haven't yet adjusted to college and the need to take responsibility for their own learning. But the old complaint - "Where will I ever use this?" - has gone away, and students view the courses as important to their future work. Because we're not trying to cram so much into the courses, there is time to make sure those who are working (which seems to be a higher proportion than in the old college algebra course) actually understand what we're trying to teach. Faculty seem to find these new courses less painful to teach because it's easier to interest students in the material.

Advising issues

The head of our advising center was very worried when we added these new courses, because many students come in undeclared or change their major in their first year or two. We've worked hard with the advising center to minimize situations in which students need to take an additional mathematics course if they change majors. If students come in genuinely undeclared (rather than simply vacillating between two

majors), we suggest that they wait a semester before taking mathematics (unless they place into calculus, since all majors except elementary education will accept calculus as a substitute for their requirement). The education school (with the mathematics department's support) won't accept anything else as a replacement for the course we developed for future elementary teachers. We require students to be sophomores for that course, since by then they have started to think of themselves as teachers as well as students and take it more seriously. Thus, those undeclared majors who think they might teach at the elementary level are encouraged to wait until their sophomore year.

We give special help to the small number of students who start out majoring in the social sciences or biology and then change their major to a subject requiring calculus (mathematics, chemistry, computer science, software engineering). The two modeling courses are acceptable as replacements for each other. Because students get a course oriented toward their current interests, they take it more seriously than they would a course unrelated to their proposed major. Yet if they later decide to change majors, their graduation isn't delayed by the mathematics course they took.

My department feels strongly that a course emphasizing symbol manipulation, rather than concepts, is an inappropriate terminal mathematics course. Therefore, we removed the standard college algebra from the list of courses satisfying the mathematics component of the university's general education requirements. It still yields three credits toward the total number needed for graduation, but it doesn't satisfy the mathematics requirement. This got the attention of advisors fairly quickly and made implementing these changes relatively simple. In addition, the department secretary screens the list of students registered for college algebra and telephones all those whose major doesn't require calculus to warn them that college algebra does not meet the general education requirements.

Scheduling issues

One potential problem with having a range of entry-level courses is that it's much easier to fill students' schedules if there are many sections of a given course. There are a variety of ways one can partition the students flowing through college algebra or precalculus, depending on institutional enrollment patterns. Of the various college algebra alternatives we offer, only the course for biology majors has fairly few sections—three per year. To ensure that these fill and are offered at times biology students can take them, we schedule them in consultation with the chair of biology. For all other courses we have sufficiently many students to fill at least three sections per semester, and scheduling them has not been a problem.

Articulation issues

After our Undergraduate Studies Committee approved these new courses, I contacted our three main twoyear colleges that feed into Monmouth University. I asked the chairs of their mathematics departments if I could visit them to discuss the changes we were making and explore what could be done in terms of their students who planned to transfer here. Of the three, the closest one was initially rather hostile to the new courses, since we are only one of three major client schools for their graduates. However, within a year they had developed a version of our social science course that meets our requirements and also meets the requirements of the main state university (Rutgers) where many of their students continue. One of the other two feeder schools already had a course similar to our social science version, and the last one was interested in at least trying one out. Two of the three schools were also interested in what we were doing with the elementary school teachers. They were not happy about our plan not to give general education credit for college algebra. On further investigation, it turned out that the lock-in articulation agreement to which Monmouth University has agreed promises that, if their students complete an associate's degree at their school, then their college algebra course will meet Monmouth's general education requirements. However, our students can't simply take a course at another school to get around our requirement—the agreement only applies to students who complete a full associate's degree. In retrospect, it would have been better if I had contacted our neighboring two-year schools earlier in the process and involved them in the course development. I was new to an institution with a large transfer population, and this is one mistake I won't make again! But the chairs of the other schools were appreciative of my taking the time to visit them to discuss what we were doing.

Another way to cut the pie

Portland Community College has split college algebra into three versions: College Algebra for Liberal Arts, College Algebra for Business, Management, Life and Social Science, and College Algebra for Math, Science and Engineering, which have the same course number, but a terminal letter, A, B or C, to distinguish among them. All three courses cover linear functions investigated graphically, numerically, symbolically, and verbally as well as logarithmic, exponential, polynomial and rational functions. They differ primarily in the applications considered. The business and science versions also include solving systems of equations. College Algebra for Liberal Arts is considered a terminal course, but the other two are accepted interchangeably as the prerequisite for Elementary Functions. This latter leads to the standard calculus sequence, to Calculus for Management, Life and Social Science, as well as to Discrete Math I. Presumably most students planning to major in business or the social sciences start in College Algebra for Business, Management, Life and Social Science, while those going into the sciences start in College Algebra for Math, Science and Engineering. If they change their major, they can still continue without retaking a course. All three versions were designed to be transferable and are accepted by the Oregon universities. The state has guidelines that say what must be taught in a course in order to give it a specific title, for example College Algebra. These guidelines are fairly broad—thus allowing for the variants Portland Community College has developed—and consequently courses vary between institutions, although they all include the same core material. College Algebra for Liberal Arts fulfills the college-level mathematics requirement at the other universities, but may transfer in with a different title. The other two versions are accepted as college algebra.

Summary

Splitting college algebra into several courses more relevant to students' majors requires an understanding of the needs of the particular institution. It also requires a few days of effort talking with client disciplines. However, the time thus spent does wonders for the image of the mathematics department at the institution and results in much less grief for faculty teaching these courses, as students are more motivated to learn the mathematics involved. And, in the long run, it should also result in a less math-phobic and antipathetic public.

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