Calculator Drill

Purpose

The purpose of this lab is to let the student know what basic calculator skills are required in this course and to provide some practice with those skills. This lab is meant to be done by students individually.

Preview

You are working along in the middle of a challenging test, which is based on a calculus lab. To answer an important question, you have to compute the expression

$$1+\left(-\frac{x^2}{2}\right)+\frac{\left(-\frac{x^2}{2}\right)^2}{2!}+\frac{\left(-\frac{x^2}{2}\right)^3}{3!}+\frac{\left(-\frac{x^2}{2}\right)^4}{4!}+\ldots+\frac{\left(-\frac{x^2}{2}\right)^{23}}{23!}$$
, for $x=0.3,\,1,\,$ and 2.5 .

Can you do it in a reasonable amount of time?

Type of Calculator

Students are required to use a TI-83 calculator for this course. For those who already own another calculator and who strongly prefer to use that calculator a waiver of this requirement may be granted upon petition from the student and successful completion of a calculator skills test.¹ Those who are considering using a calculator other than the TI-83 should have their own calculator manuals. If you qualify to use a different calculator, then you should not expect to receive any technical assistance concerning your calculator from your lab instructor (of course, if your instructor happens to know your calculator well, then you will be able to get some help). In any case you should be able to perform all of the skill exercises in this *Calculator Drill*.

Suggestions for Working this Lab

If a task seems to be longer than it should be (for example, adding two long lists of numbers term-by-term), then you should try to find a shortcut (for example, your calculator probably has functions that operate on all elements of a list at once). Process is more important than answers in this lab. Seek all the help you need (maybe another student in the class has the same model calculator, or maybe there is a helproom worker who is familiar with your calculator), but make sure that you can carry out all of the procedures in this lab on your own. You won't have time to figure

¹If you want to petition to use a different calculator, you must ask your teacher or your lab instructor for the calculator test. You will take the test on your own time and submit it to your lab instructor to be graded. If your performance is sufficient, then you will be allowed to use your "other" calculator in the labs. If you don't do well on the test, then you will be required to get a TI-83.

out these calculator skills when you need them in later labs, where your primary concern will be mathematical concepts.

Part 1: Function Plotting

- 1. Plot the function $f(x) = x^3 9x^2 48x + 52$, with horizontal and vertical ranges set to [-10,10].
- 2. Using the same horizontal range, replot the function, f, but set the calculator to scale the vertical axis automatically. If your calculator does not automatically erase the previous graphing screen, be sure you do so manually before you make another graph, unless your intention is to superimpose graphs.

With the graph of f on the screen, practice zooming. Zoom in, and zoom out. Zoom in on interesting features of the graph. Put a portion of the curve in a zoom box, and zoom in on that box.

- 3. Put your calculator in radian mode^2 , and superimpose the graphs of $f(x) = \cos(x)$ and $g(x) = \sec(x)$ over the interval [-10, 10]. (Most calculators don't have a \sec key, but you can make use of the trig identity $\sec(x) = \frac{1}{\cos(x)}$.) If you have plotted these graphs correctly, you will notice an interesting relationship between the graphs of f and g.
- 4. Make a graph of $f(x) = 2\cos(x-1) + \sqrt{5}\sin(x-1)$ over the interval [-3,4]. Use the "trace" capability of your calculator to move the cursor along the curve, and place the cursor on one of the highest points (locate the point by sight and by noting the coordinates of the cursor). Record the coordinates of this point.

Your calculator should also be able to find this highest point automatically. (There is usually a key or a menu choice such as *maximum*, *MAX*, *extremum*, *or EXTR*.) Have it compute the maximum now. The calculator likely did not produce exactly the same answer that you did manually. Can you explain how such a discrepancy could happen?

5. On the graph from step 4, trace along the curve to a root of the function, and record the coordinates from the screen. Then have your calculator compute the same root with its built-in root finder, and explain any discrepancies between the answers.

²In calculus courses we almost always use radian mode when we work with trig functions. You should assume that radian mode is the appropriate mode, unless it is explicitly stated otherwise.

Part 2: Solving equations and finding roots

1. Your calculator should have a routine for solving equations. Have your calculator solve the equation

$$4.9t^2 - 18 = 0$$
.

Check the answer by solving the equation by hand.

2. Solving the equation, $4.9t^2 - 18 = 0$, is equivalent to finding the roots of the function

$$f(t) = 4.9t^2 - 18.$$
 [Do you know why?]

Graph the function, f, and then use your calculator's built-in root finder to find the roots of f. There are two roots of this function, and one advantage of working from the graph is that you may be able to see approximately where the roots are and how many there are before you find them accurately. Of course, whether you see all the roots of a function depends upon the ranges shown in the graph.

Part 3: Data Entry and Manipulation

In your calculator store under the three labels, \mathbf{t} , \mathbf{d} , and \mathbf{v} , the data from the table on the next page. You are going to be doing several operations with this data, so be sure you store it in the right format; for example, your calculator may have choices such as a *list*, an *array*, or a *matrix*. It is possible that you may need one format for one operation, and another format for a different operation; thus, it can useful if you learn to change data from one format to another. For the purposes of this lab, we will refer to each of the three stored sets of numbers as *lists*.

t	d	v
1.2	7.056	11.76
3.4	56.64	33.32
4.1	82.369	40.18
4.9	117.649	48.02
5.3	137.64	51.94
7.1	247.009	69.58

- 1. Make two scatter plots, one for each of the following pairs of lists:
 - (a) **v** against **t** (i.e., put **v** on the vertical axis, and **t** on the horizontal axis).
 - (b) v against d.

84 Lab: Calculator Drill Duke University

- 2. Create a new list, each entry of which is equal to the corresponding entry of \mathbf{v} divided by the corresponding entry of \mathbf{t} . Be sure you use the list features of your calculator to do all these calculations at once—don't do them one-by-one. If you did this correctly, then there will be a noticeable feature of your new list.
- 3. Make a list whose entries are the differences of consecutive elements of the list **d**. We will denote the new list by $\Delta \mathbf{d}$. (Your calculator may have a single key to accomplish this operation.) In a similar way construct a list, $\Delta \mathbf{t}$, from **t**. Now compute the list, $\frac{\Delta \mathbf{d}}{\Delta \mathbf{t}}$. The final list should have five entries, the first of which is about 22.5382.
- 4. Make a scatter plot of **d** against **t**. If your calculator gives you a choice, plot the points with large dots, or \times 's, or anything distinguishable from the usual points. Superimpose a graph of the function $s(t) = 4.9t^2$ over your scatter plot. Be careful not to change your ranges from one plot to the next, else you will not get an accurate picture. You should see a good "fit" of the function to the data.

Part 4: Defining and Working with Functions

You should be able to define a new function in your calculator. For example, consider the function $f(x) = \sqrt{x-1}$. If you need to compute many values of this function, and make a graph of f, and use f(x) in other expressions, then it can be useful (or necessary) to set up in your calculator a key or a short expression that will return values of f.

On some calculators you can assign a formula to a built-in function name, such as Y_1 , so that evaluating $Y_1(x)$ would return the appropriate result, if x has been assigned a value. On other calculators there is a "Define Function" key, $\overline{\rm DEF}$, which takes a formula such as $f(x)=\sqrt{x-1}$, and creates a user key (or *soft key*) that can be used as a built-in calculator button.

- 1. Define the function $f(x) = \sqrt{x-1}$ in your calculator, and then test it by computing f(1), f(3), and f(5). To practice using the function in another expression, compute sin(f(7)). The answer is about .6382.
- 2. In some of our labs you will need to define a function, and then use that function in the definition of another function. Define the function $f(x) = .2x^3$ on

your calculator. Now, using the name or symbol you assigned that function, define this function:

$$g(x) = \frac{f(x+.01)-f(x-.01)}{.02}$$

The idea here is not to simplify the expression before entering it into your calculator, but rather to use the function you've already stored in the definition of g. Check to see that you can get your calculator to compute values of g correctly by testing the values g(0), g(2), and $g(\pi)$. The answers are .00002, 2.40002, and 5.92178264 respectively.

3. Make a graph of g on your calculator. If you have it right, you should see a parabola.

Part 5: Sums of large numbers of terms.

There will be a number of occasions when we have to compute a large, iterative sum such as the one below:

$$1^3 + 2^3 + 3^3 + \dots + 1500^3 = \sum_{k=1}^{1500} k^3$$

On some calculators you have to set up a sum of an iterated sequence. Others will accept a shorthand like that above.

- 1. Find out how to enter a large sum on your calculator, and then compute the sum above. The answer is 1267313062500.
- 2. Compute the sum, $\sum_{k=1}^{1100} \frac{(-1)^{k+1}4}{2k-1}$. The answer is close in value to a well-known, irrational number.
- 3. Recall that 5! is defined to be $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$. Your calculator can probably compute this number, but the factorial key may be hard to find. It is often placed in a menu for *probability* operations. You will need this factorial key to set up the sum $\sum_{k=0}^{50} \frac{1}{k!}$ on your calculator. Compute this sum, which you will see approximates another well-known constant.

4. On your calculator define $f(x) = \sum_{k=0}^{4} \frac{(-1)^k x^{2k+1}}{(2k+1)!}$. (Don't convert the expression to

longhand. Learn how to set up an expression like this one on your calculator.) Now graph f with horizontal and vertical ranges of [-6.3, 6.3] and [-3, 3] respectively. The middle part of the curve you see will resemble a familiar function.

Part 6: Optional, advanced calculator skills.

For students who are adept at using programmable calculators, here are a couple of exercises that require more advanced skills that may be useful.

1. Have your calculator graph a piecewise defined function such as the following:

$$f(x) = \left\{ \begin{smallmatrix} -x, \text{ for } x \leq 0 \\ x^2, \text{ for } x > 0 \end{smallmatrix} \right.$$

2. Write a short program that computes a list of values based on a recursive rule. Your program must be given the first entry, denoted here by A(1), and then it would compute the other entries according to a recursive rule such as the following:

$$A(k+1) = k \cdot A(k)$$
, for $k = 1, 2, ..., 50$.