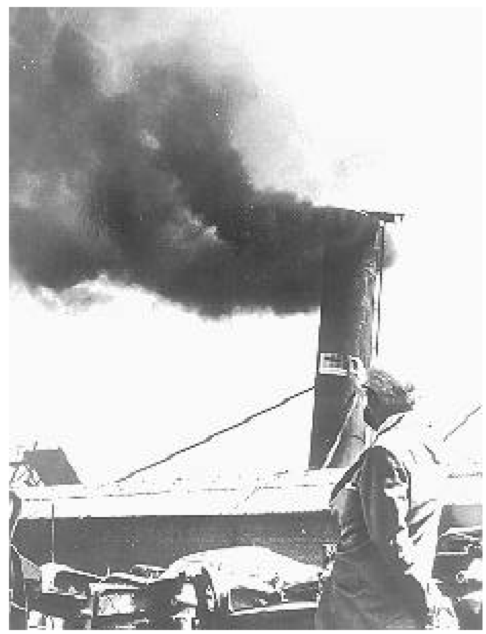
Air Pollution: Particulate Matter

Purpose

We will use this lab to develop an understanding of how fine, airborne, particulate matter is measured by industry and regulators. We will use functions and integrals to understand how aerosol concentrations are modeled, and we will use numerical methods of computing integrals to make the connection between the theory and data.

Preview

Scientists, public health advocates, environmentalists, and Environmental Protection Agency regulators are concerned with airborne fine particulate matter (PM) for two reasons: PM is one of several pollutants that combine to create smog, thereby limiting visibility and obscuring scenic vistas; and inhalation of PM contributes to a variety of respiratory illnesses.



Terminology

Although we speak of haze, dust, and smoke as different atmospheric phenomena, all of these things are caused by particles in the air[1]. The particles may be solid or liquid, and they may be spherical or irregular in shape. Nevertheless, all of these airborne elements are classified as *aerosols* by environmental scientists. Thus, we will be speaking of measuring aerosols in the atmosphere.

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[1]To see the effects of airborne particulates refer to this web page at the School of the Environment: http://www.env.duke.edu/courses/env101/env101_vis.html

Part 1: Measuring Particulates

The aerosols in the atmosphere over a city generally have a great variety of sizes of particles: they typically range from 10 angstroms ($10\,\text{Å} = 10 \cdot 10^{-10}\,\text{m} = 10^{-9}\,\text{m}$) to 100 microns ($100$ microns $= 100 \cdot 10^{-6}$ meter $= 10^{-4}$ meter) in size. But the effect on our lives is not simple. Indeed, it is typically neither the largest nor the smallest particles which are of most concern in our breathing; rather there is an "optimal" range of sizes, which is a mid-sized range, that causes the greatest problems for our lungs. This issue of the size of the particles has motivated scientists to use the diameter of the particles both in their measurements and in their analyses.

If a particle is spherical, "diameter" makes sense. For irregular particles the notion of diameter has to be defined carefully. In this case scientists define the diameter of the particle to be that which a spherical particle of the same substance would have to have to exhibit similar atmospheric properties.

Exercise: Suppose a particle is in the shape of cube, which measures 1.2 microns on each side. For this exercise assume that the volume is the only significant environmental property of the particle. What is the "diameter" of this particle?

Part 2: The Size Distribution Function

Because the number of different diameters of particles in a polluted atmosphere can be so large, scientists assume that the variation in diameters of particles is continuous. When scientists make measurements of aerosols, they can measure, for a typical cubic centimeter of air, the number of particles that fall within a small range of diameters. This method of measurement is more efficient and practicable than trying to measure the quantity of every size particle present.

Let p represent the diameter of particles in an atmosphere. Using the previously stated variation in sizes, we note that typically

$$10\,\text{Å} \leq p \leq 100\,\text{microns}\ .$$

Let $N(p)$ be a function that expresses the number of particles of diameter less than or equal to p, contained in a cubic centimeter of air over a fixed location. The derivative of this function, $\frac{dN}{dp}$, expresses the rate of change in the number of particles (per *cm*^{3}) as the value of p changes; $\frac{dN}{dp}$ is called the size distribution function for aerosols. The approximation of this derivative, $\frac{\Delta N}{\Delta p}$, would involve the measurement of how many particles in the size range $[p, p + \Delta p]$ are in a cubic centimeter of air. Here we have the link between the continuous model and the data.

What scientists can measure is ΔN for various intervals of p. This measurement is practicable because a range of sizes, rather than a particular size, can be filtered and measured. After scientists have the measurements of ΔN, they can compute and tabulate values of $\frac{\Delta N}{\Delta p}$. If we have values of $\frac{\Delta N}{\Delta p}$, then using the approximation $\frac{\Delta N}{\Delta p} \approx \frac{dN}{dp}$, we can estimate the number of particles in a cubic centimeter of air.

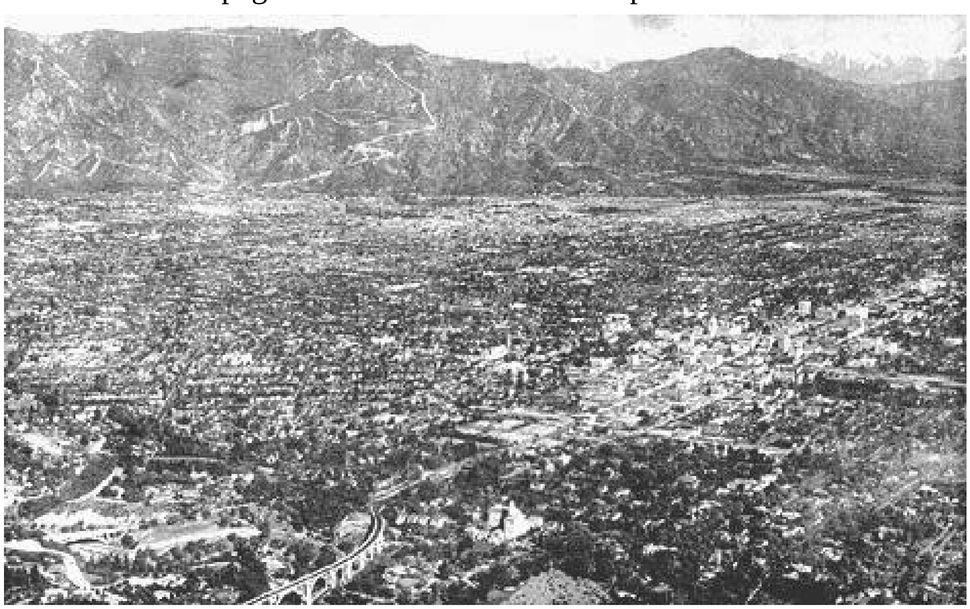
1. Explain the meaning of the integral $\int_{p_1}^{p_2} \frac{dN}{dp}dp$.

2. When the EPA sets a standard for fine particulate matter being emitted into the atmosphere, the Agency specifies a diameter, p; then any emissions must be filtered for particles of diameter greater than or equal to p. Write down an integral that represents the number of particles that the current standard of 10 microns would cause to be removed from the air.

3. Based on emerging scientific evidence that the smaller particles are dangerous because they lodge deep within the lungs, the EPA announced in the spring of 1997 its intent to lower the regulatory standard from 10 microns to 2.5 microns. Write down an integral that represents the number of additional particles (*per cm*^{3}) that would be removed if the standard were changed from 10 microns to 2.5 microns.

Part 3: Working with Real Data

The data on the next page were taken from the atmosphere over Pasadena.



Pasadena, California

The unit of measure for the diameters, p, is *one micron*, and the unit of measure for $\frac{\Delta N}{\Delta p}$ is *number per cm*^{3} *per micron.*

p	$\frac{\Delta N}{\Delta p}$
0.00875	$1.57 \cdot 10^7$
0.0125	$5.78 \cdot 10^6$
0.0175	$2.58 \cdot 10^6$
0.0250	$1.15 \cdot 10^6$
0.0350	$6.01 \cdot 10^5$
0.0500	$2.87 \cdot 10^5$
0.0700	$1.39 \cdot 10^5$
0.0900	$8.90 \cdot 10^4$
0.112	$7.02 \cdot 10^4$
0.137	$4.03 \cdot 10^4$
0.175	$2.57 \cdot 10^4$
0.250	$9.61 \cdot 10^3$
0.350	$2.15 \cdot 10^3$
0.440	$9.33 \cdot 10^2$

p	$\frac{\Delta N}{\Delta p}$
0.550	$2.66 \cdot 10^2$
0.660	$1.08 \cdot 10^2$
0.770	$5.17 \cdot 10^1$
0.880	$2.80 \cdot 10^1$
1.05	$1.36 \cdot 10^1$
1.27	5.82
1.48	2.88
1.82	1.25
2.22	$4.80 \cdot 10^{-1}$
2.75	$2.17 \cdot 10^{-1}$
3.30	$1.18 \cdot 10^{-1}$
4.12	$6.27 \cdot 10^{-2}$
5.22	$3.03 \cdot 10^{-2}$

Use the data, integral calculus, and numerical methods of integration to answer the following questions.

1. Set up an integral that represents the total number of particles per cubic centimeter found in the atmosphere above Pasadena. Estimate the value of this integral, and explain how you made the estimation. Is your estimate an over or under estimate?
2. How many particles per cubic centimeter lie between .05 microns and .55 microns?
3. Under the proposed 2.5 micron standard, all particulate matter of diameter 2.5 microns and above should be filtered out of the atmosphere. What per cent of the total number of particulates would have been removed had this standard been followed in Pasadena? What standard would be required if we wanted to reduce the number of particulates by 10%?

Part 4: Computing Mass

When scientist make recommendations concerning environmental standards, the most important single measure is the mass of the pollutant that is present. If we have data on the number of particles per cubic centimeter for some particle diameter p, and since we can compute the volume of each particle, then knowing the mass density of the particle would enable us to compute the total mass of particulates. The actual density of particulate matter varies slightly with the different types of particles, but a typical density is about 1.5 $\frac{g}{cm^3}$. Use this value for density in answering the questions below.

1. Write down an expression for the mass of a single particle of diameter p microns.

2. Set up an integral that represents the total mass of particles per cubic centimeter found in the atmosphere above Pasadena. Estimate the value of this integral, and explain how you made the estimation. Is your estimate an over or under estimate?

3. Write down an integral that represents the additional mass of particulate matter (per cm^3) that would be removed from the air over Pasadena if the standard were changed from 10 microns to 2.5 microns. What standard would be required if we wanted to reduce the mass of the particulates by 10%?

Report

Your report should include written answers to all the questions above. Compare the answers to the questions in step 3 of both Part 3 and Part 4. Explain (or show) all computations clearly.