

## Math 32L First Day Handout Fall Semester

The following exercises represent some of the most important concepts from first-semester calculus. Students who are beginning Math 32L should be familiar with these concepts, and the problems that are presented here are meant to serve as a review of those concepts. Work together in groups of three or four students to complete the statements or answer the questions. Please hand in this sheet at the end of class or at the beginning of the next class, as directed by your teacher.

Names: \_\_\_\_\_  
 \_\_\_\_\_

1. The derivative of  $f$  at  $x$ ,  $f'(x)$ , is defined to be  $\lim_{h \rightarrow 0}$

2. The definite integral of  $f$  from  $a$  to  $b$ ,  $\int_a^b f(t) dt$ , is defined as follows:

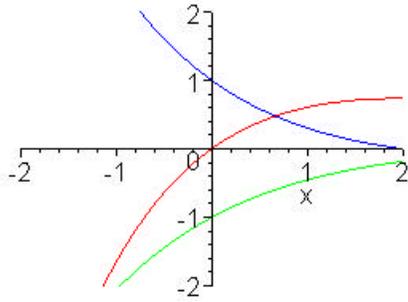
$$\int_a^b f(t) dt = \lim_{n \rightarrow \infty} \sum_{k=1}^n \quad , \text{ where } \Delta t = \frac{b-a}{n} \text{ and } t_k = a + k \Delta t.$$

3. When we evaluate  $\int_0^2 x^3 dx$ , we find an antiderivative of  $x^3$  and then use this antiderivative in the computation of the value of  $\int_0^2 x^3 dx$ . But that is very different from the description of the definite integral described above in question 2. What theorem describes the relationship between definite integrals and antidifferentiation and what does it say?

4. Let  $F(x) = \int_3^x f(t) dt$ . Which two expressions below are equivalent to  $\frac{F(x+h) - F(x)}{h}$ ? Assume that  $h$  is a small, positive number.

$F'(x)$	$F(x)$	$f(x)$	$f'(x)$
an approximation of $F'(x)$	$F(3)$	an approximation of $f(x)$	
area under $F$ over $[x, x + h]$	area under $F$ over $[3, x]$	area under $f$ over $[x, x + h]$	

5. On the graph below identify which graph is  $f$ ,  $f'$  and  $f''$ . Explain your answer in few complete sentences.



6. Find the linear approximation of the function  $y = xe^{kx}$  at  $x = 0$ , where  $k$  is some positive number. Does the linear approximation underestimate or overestimate  $xe^{kx}$  near 0? Explain your answer carefully.

7. Experience suggests that the rate at which people contribute in a charity drive is proportional to the difference between the current total and the announced goal. A fund drive is announced with a goal of \$90000 and an initial contribution of \$10000. Write an initial value problem (i.e., a differential equation with an initial condition) to describe the total amount of money raised (as a function of time). You do **NOT** have to solve the differential equation.

8. Circle all of the following expressions which are solutions of the differential equation  $\frac{dy}{dx} = y - 2x$ .

(a)  $y = 2x + 2 + e^x$

(b)  $y = 2x + 2 - e^x$

(c)  $y = e^x - x^2$

(d)  $y = e^x - x^2 + 1$

9. Suppose  $f(x) > 0$ ,  $f'(x) < 0$  and  $f''(x) > 0$  for all  $x$ . List the following quantities from the smallest to the largest. LHS refers to the left hand (Riemann) sum; RHS to the right hand (Riemann) sum.

$\int_a^b f$     LHS,  $n = 10$     RHS,  $n = 10$     LHS,  $n = 20$     RHS,  $n = 10$

smallest: \_\_\_\_\_ largest