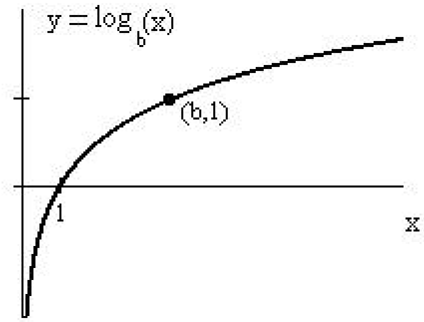
Properties of Logarithms

Assume that b is a constant greater than 1. Let $y = \log_b(x)$. This logarithm function is by definition the inverse of the function $y = b^x$. The domain of $y = \log_b(x)$ is $(0, \infty)$. The range is $\mathbb{R}$.



In the statements below, assume that x and y are arbitrary positive numbers.

1. $\log_b b = 1$

2. $\log_b 1 = 0$

3. If $0 < x < 1$, then $\log_b x < 0$.

4. If $x > 1$, then $\log_b x > 0$.

5. $\log_b(xy) = \log_b x + \log_b y$

6. $\log_b \left(\frac{x}{y}\right) = \log_b x - \log_b y$

7. $\log_b x^p = p \log_b x$

8. $\log_b x = \frac{\log_B x}{\log_B b}$

9. $\log_b b^x = x$, for all x.

10. $b^{\log_b x} = x$, for all $x > 0$.

Notes:

1. In the case the base, b, is the number e, we write $\ln x$ for $\log_e x$. A logarithm with base e is called the "natural logarithm" for reasons we'll see later in the course.

2. The base of a logarithm is usually chosen to be greater than 1; however, any positive constant other than 1 can be used. If the base b is between 0 and 1, then the graph of $y = \log_b(x)$ will look like the one shown to the right.