Fall 2007 Math 25L Syllabus
Instructor's Version
Textbook: Calculus (4th ed), by Hughes-Hallett, et al.

General comments on the first week: Students will think they know this material, and you will bore them by trying to lecture on all of the topics in the first chapter. We recommend that you lecture on selected topics only, and spend some time having them work in groups on challenging problems based on the material they are reading.

Day	Section	Topic	Homework

1-1 Cspk. Algebra Expectations, Introduction Cspk. p27/1-21.

Notes: Don't try to cover this material in the classroom. It should be an easy review for students, so you can talk about your policies, etc, and leave the assignment for the students to do on their own. Briefly describe the course, labs, and grading policies. You should not make a big issue out of this course being different from the traditional course; simply tell students that it emphasizes applications and why things work. Give out Departmental and individual handouts. Also, tell students about information on the web: they can find math placement information at *www.math.duke.edu/first_year*, as well as links to course home pages, help room schedules, etc. Tell students that a TI-83 calculator is highly recommended for this course. TI-84, 85, and 86 are acceptable, but all calculator exercises in lecture and lab will be done using the 83. They need to have their calculator for the first lab on Thursday. Make sure they can read the syllabus and point out that the coursepack is also the lab manual and it changes each year. Tell them when and where the lab will meet.

1-2 1.1, Cspk Functions 1.1/1-3,15-27,32; Cspk. p31/1-7.

Notes: Most students think of functions as simply formulas. After you describe the nature of a function, you will want to show them that functions can be represented by formulas as well as graphs, tables and verbal descriptions. Use examples to cover domain, range, dependent and independent variables. Do not spend time finding the ranges of functions algebraically. This can be done when we cover inverse functions in week 8. Have them find the range of functions such as $f(x) = x^2 + 1$.

Lab: Using Your Calculator

Notes: This is an easy but important lab. The students will go over all the calculator commands needed for both 25L and 26L. It is important for you to use the calculator throughout the semester when working examples or explaining concepts in lecture. The students should understand that we expect them to handy with their calculator. There will not be a quiz or lab report for this lab.

1-3 1.1 Linear Functions 1.1/4-14,29,31,33,34,38,39; Cspk. 33p/1,2.

Notes: Students should know: given two points, how to find the equation of the line using $y = mx + b$ or $y - y_1 = m(x - x_1)$; parallel lines have the same slope and perpendicular lines have negative reciprocal slope; given the equation of the line, identify the slope and y-intercept; interpret slope as a rate of change. The Olympic high jump example in the text is very nice.

Day	Section	Topic	Homework

2-1 1.6,Cspk. Power Functions and Polynomials 1.6/5,7,10,12,13,16-18,20-22,25,26,32; Cspk. p33/3; Cspk. p35/1,2.

Notes: Show them the difference between the graphs of $y = x^{\text{even power}}$ and $y = x^{\text{odd power}}$. Have them experiment with their calculators. The students should be able to quickly graph a factored polynomial by plotting its intercepts, then considering the multiplicity of it roots. Ideas to bring into the mix, odd and even functions, even degree polynomials leave through either the 1st and 2nd quadrants or 3rd and 4th, for odd degree polynomials it is 1st and 3rd or 2nd and 4th, and conditional functions (Cspk. problem 33/3).

2-2 1.6,Cspk. Rational Functions 1.6/6a,b,14,28,29; Cspk. p36/3-5.

Notes: Use the ideas developed for graphing factored polynomials to graph factored rational functions. The multiplicity of the roots in the numerator characterize the graph near its x-intercepts. Before covering vertical asymptotes you will need to get across the idea that

as the denominator shrinks (and the numerator does not) the function approaches $\pm\infty$. Finding horizontal asymptotes by comparing the degree of the denominator and numerator. Let them know why it is so difficult to graph rational functions on their calculators(scaling problems).

Lab: Linear Modeling

Notes: This is an involved lab that has the students model the velocity of a free falling object using difference quotients and a line of best fit. Familiarize yourself with this lab. You can use the ideas presented in the lab in lectures on 3-1,3-2 and 3-3. You will most likely require a write-up for this lab.

2-3 1.3,Cspk. New Functions From Old 1.3/5-8,14,28-34; Cspk. p39/1-6.

Notes: This is a great opportunity to have students work through some examples with their calculators so that they can compare, for example, $f(x+3)$ and $f(x)+3$. You can have the students enter a function into y_1 on their calculators then enter $y_2 = y_1(x+3)$ and $y_3 = y_1(x)+3$. Point out when one flips and translates the graphs of functions the order of the transformations matter. Function composition is covered in week 6.

3-1 2.1 Rate of Change 2.1/1,2,5-7,9,11-17.

Notes: You may want to have students work along with you on an exercise. You can ask them to compute some average velocities with their calculators over shrinking intervals of time. Talk about the problem of observing instantaneous velocity and why we need the concept of a limit. Use the linear modeling lab as an example.

3-2 2.2 Derivative at a point 2.2/1,7-11,14,15,21,33,35,38,39,42;

Notes: Most of your students have had high school calculus. They think they understand these topics, but many don't. Make them compute the derivation at a constant value of x by using the definition.

Lab: Strategies for Buying Stock

Notes: The idea of this lab is to reinforce the meaning (and usefulness) of difference quotients (and second difference quotients). This lab has a very nice write-up. The students pick a stock buying and selling strategy and try them out a a real stock.

Day	Section	Topic	Homework
3-3	1.7,1.8	Limits	1.7/1-6,15,17,18,22;1.8/1,15,19-21,27,28, 37,39,42;begin day 4-2 review.

Notes: After the previous lesson, the students should sense that limits are important, that they are the "bridge" between precalculus and calculus. Do not try to do an ϵ-δ presentation. Use intuition, graphs, and numerical approximations. Tell students that they are NOT responsible for the ϵ-δ definition and the corresponding examples in the textbook. Be sure to warn students to start doing the review problems (day 4-2 HW) this weekend because the first major test is coming up next week.

4-1	2.4	The Derivative Function	2.4/1-3,5,8,11,13,15,18,21,25,29,32,33,35,40,42; finish day 4-2 review problems.

Notes: Try to convince the students that the derivative is a function and that it means something. To students who took high school calculus it appears to be only a "formula." Check out #32, looks ahead to the derivative of e^x.

4-2	*Review*	p58/1,8,11,14,15,17,18,24,25,32,34,35,37,45,46; p103/1,3,7,9,10,12,14,15,18,19,21.

Lab: Test #1

Note: If you're new to Duke, be sure you look at some experienced instructors' old tests before you write your test. A number of copies of old exams are in the resource room (118 Physics). It would be preferable for you to proctor your own test, but if you can't be in lab, then your lab assistants will have to administer the test; however, you, not your lab assistants, should grade the test.

4-3	2.4	Interpretations of the Derivative	2.4/1-3,5,7,9,10,14,16,18,19,21.

Notes: Lead the class through a careful discussion of some examples. The students are not used to thinking like this—especially those who recite the phrase "instantaneous rate of change." Example 2 on page 90 is a good example of the type of interpretations you should be presenting. Draw some pictures and tie these interpretations to linear approximations in general with $\Delta t = 1$.

5-1	2.5	Second Derivative	2.5/1-5,7,9,12,15,16,18,20-22.

Notes: Recall that students have encountered this idea in the lab on Stock Prices. Most will know the concavity interpretation from high school, but be careful in your explanations, because they won't understand why the "rules" they've memorized are true. They are also unlikely to have seen an interpretation of the second derivative other than in the context of acceleration of a moving body or concavity of a curve.

5-2	2.6	Cont. and Differentiability	2.6/1-7,10,13.

Notes: Here's where you can bring up some of the problem cases—but don't use ϵ-δ arguments. Draw pictures and use intuition and do some calculations with the calculator. A good calculator-based demonstration is to have students graph $y = |x|$, and then zoom in on the origin several times. Some students will think their calculators are not zooming in correctly!

Lab: Binomial Theorem Lab

Notes: This lab contains material that is not directly related to the topics you are currently covering. The lab concludes with the binomial expansion of $(x+h)^n$. You will be able to use this when you present the "power rule" for derivatives.

Day	Section	Topic	Homework

5-3 3.1 Sum, Difference & Power Rules 3.1/1,2,4,6,7,10,11,16,20,21,30,33,41,43,53, 54,56,58,60,61,63,65,70.

Notes: Don't forget that the students have been through the binomial theorem lab. Look at the homework problems and spend some time working an example or two like the hardest ones. If time permitted, the students may have worked through a proof of the power rule at the end of lab. Contact the lab TAs. In either case, work through a "proof" of the power rule with a specific exponent, like 50.

6-1 3.3 Product and Quotient Rules 3.3/1,8,15,16,19,20,23,24, 30-32,34,37,40,45,46,49,50,53,54,58.

Notes: Easy stuff. Have the students work an example so they can see that it cannot be true that $(f \cdot g)'$ is the same as $f' \cdot g'$. You can show them that the units of $f' \cdot g'$ are not the units of $(f \cdot g)'$. The book has a slick way of deriving the quotient rule.

6-2 1.3,Cspk. Composition 1.3/43-46,51,54;Cspk. p41/1-5.

Notes: This seems like easy stuff but many students confuse composition with multiplication. Give a short lecture, and then have students work together in class on some hard problems at the end of the section. You may be surprised at how hard some students find these problems.

Lab: Introduction to Euler's Method

Notes: This topic is new to most students, even those who have had a lot of high school calculus. It's here because it's a nice application of the derivative, and using and understanding Euler's method requires students to understand the derivative and basic ideas about slopes of lines. After the lab is over, you can make the connection between this application and the linear approximations. Furthermore, this method will, later in the course, give us an excellent way of arguing (not a formal proof, of course) the validity of the Fundamental Theorem of Calculus.

6-3 3.4 Chain Rule 3.4/1,3,7,9,23,34,49,53,54,57,60,61,69,71-74,84; Cspk. p43/1-6

Notes: With some carefully drawn pictures you can show them where the multiplication comes from in the chain rule. Work a variety of examples. Make sure you read over the problem in the Cspk. By careful consideration of units you can convince the students that $(f \cdot g)' \neq f' \cdot g'$.

7-1 *Fall Break*

7-2 Differentiation practice. Text p159,160/2,17,36-38,41,51,58,65,66,72,81-83; Cspk. p45/all.

Notes: This day allows everyone to catch their breath and practice differentiation with the different techniques. It is a good day for a quiz. You can have students work on groups on problems.

7-3 1.2 Exponential functions 1.2/5,6,13,14,17-20,22,30-32,34,36,37-39.

Notes: A good classroom exercise is to give the students a table of data (which is an exponential function of time—but don't tell them) and ask them to compute ratios of consecutive values. Then show them what that means. The population of Mexico taken from the book works fine. Problem # 20 is very tricky.

Day	Section	Topic	Homework

Lab: Compound interest

Notes: This is an easy lab that does not lend itself to a report. The students see e for the first time.

8-1 1.3 Inverse Functions 1.3/21-27,40,42,52,53;Cspk. p47/1-5

Notes: You can revisit the definition of a function and the vertical line test. Show them how you can compute the range of a function by finding the domain of its inverse. Work through an example like $f(x) = \frac{2x-1}{3x+1}$. Discuss what it means for a function not to have an inverse (which is a function). Give examples which include units.

8-2 1.4 Logarithms 1.4/2,7-10,18-20,28,32-34,36,38,39,40,45; Cspk. p49/1-3.

Notes: There are two lessons on logs, so there is no need to rush through the material. You can extend your lecture on inverse functions to introduce the basic properties of logs, the domain and range. Ask them why they get an error when they use their calculators to take the log of a negative number. Introduce the "laws of logs" and work examples.

Lab: Logarithmic Scales

Notes: This is relatively short, straight forward lab. It lends itself to a quiz and/or a report.

8-3 1.2,1.4 Cspk. The Natural Logarithm 1.2/1-4,23,24;1.4/1,3,5,6,13-16,23,29,30,37,43; Cspk. p51/1-4.

Notes: Since this lesson is relatively short you have time for a quiz, answer homework questions and or prove some of the "laws of logs". You can prove $\log_b(AB) = \log_b(A) + \log_b(B)$, and assign them the others.

9-1 3.2 Derivative of $y = a^x$ 3.2/1,6,14,16,21,23,25,37,38,41,43;3.3/6,12,27,52,55;3.4/11, 13,17,18,35,42,47,51,63,81.

Notes: Given the graph of $y = a^x$, have them sketch the graph of the derivative of $y = a^x$. Why is the result surprising? Then start with the difference quotient and end up by experimenting with the calculators to finish off the limit. Work a number of examples which require the chain, product and quotient rules.

9-2 3.6 Derivative of $y = \ln(x)$ 3.6/1,3,4,5,8,9,22,28,29,31,39,41-44.

Notes: Given the graph of $y = \ln(x)$, have them sketch the graph of the derivative of $y = \ln(x)$. Have them guess at the derivative from their graph. Use the derivative of $y = e^x$ to find the derivative of $y = \ln(x)$. Work examples.

Lab: Log Plots:

Notes: This is also a precalculus lab. It requires the entire period and is more complicated than the previous lab. Students have trouble grasping the reasoning behind why if a semi-log graph is linear the original data is exponential and given the equation of the semi-log line how does one find the exponential model. Same for log-log graphs and power function models. It would help to briefly revisit these ideas at the start of your next lecture.

9-3 3.9 Linear Approximation 3.9/1-6,10,12,21; begin day 10-2 review problems.

Notes: This will probably be a short lecture tying in ideas from the early derivative lectures and Euler's method.

Problem # 21 is tough, you may want to offer some guidance.

Day	Section	Topic	Homework
10-1	4.7	L'Hopital's Rule	4.7/1-3,9-15; Cspk. p53/odd; finish 10-2 review problems.
	Notes: The L'Hopital's Rule homework from the coursepack has been divided up between this lesson and the 10-2 review problems		
10-2	Review		p58/36,38,3940,41,49;p103/4-6,8,22,23,26,28,31;p159/1-3, 7-9,17,29,31,37,41,42,58,66,81-83,84&85(b,d,f),88,90,95-97,99; Cspk. 53/even.
Lab:	**Test #2**		
10-3	3.7	Implicit Differentiation I	3.7/1,5,6,8,10,11,19,23,28,29.
	Notes: There are two lessons on implicit differentiation so there is no need to rush. Some students will have a hard time seeing why the if $(x^3)' = 3x^2$ then why does $d(y^3)/dx \neq 3y^2$. It might be helpful to replace all the ys in the equations with $f(x)$ then differentiate. That seems to work. Once they get the hang of it, that first step can be skipped.		
11-1	3.7	Implicit Differentiation II	3.7/2,7,16,22,24,27,30-32,35.
	Notes: Go over homework, work along with students on some more involved problems. Good day for a quiz.		
11-2	4.6	Related Rates	4.6/19-21,23,27-29.
	Notes: Outline a general strategy for solving these problems. Make sure that you look at the problems they will be working on in lab tomorrow. Give some hints to get them started on these problems.		
Lab:	**Related Rates**		Cspk. p55/5-16.
	Notes: Students will work in groups on these problems.		
11-3	4.1	Using 1st & 2nd Derivatives	4.1/3,5,7,10,15,17-20,24.
	Notes: The next two lessons bring together a number of previously introduced concepts. You will want to draw lots of pictures to illustrate the various types of critical points (on differentiable vs continuous functions) and inflection points. Note that local extrema can be located at endpoints, e.g., 0 is a local min of $y = \sqrt{x}$. Look over homework assignments and cover what is needed to work the problems.		
12-1	4.1	Using 1st & 2nd Derivatives	4.1/6,14,27,29,32,34,38,42-44,46,47.
	Notes: Finish up what you were not able to get to in the last class. Good day to catch up on homework questions and/or have a quiz.		
12-2	Teacher's Choice		
	Notes:		
Lab:	**Derivatives and Roots**		
	Notes: This is a fairly straight forward lab. The tricky questions involve the relationship between the number of roots of a functions and the number roots of its derivative and vice versa.		
12-3	Optimization		4.3/4,5,8,15,18,19,22,30,33;4.5/11,16,17,19;begin day 14-1 review problems.
	Notes: These are challenging problems. Outline a general strategy for solving them and work through some of them. Give hints for others. At the end of each problem students should offer some sort of justification for how they know they have found the maximum or minimum (1st or 2nd derivative test is handy). Problems 30 and 33 are difficult. You may want to offer some hints.		

Day	Section	Topic	Homework
13-1	4.5	Optimization	4.5/4,18,20,21,23,25,27,32;finish day 14-1 review and begin 14-2 review problems.

Notes: Work more examples of optimization problems. Feel free to leave out some of the details of the solution so that the students have to work through the problem. For example, "now that we have these two equations solve for w in this equations and substitute it in to that equation. When you differentiate, set it equal to 0 you should get $l = \sqrt{5}$. Don't forget to justify your answer."

13-2 *Thanksgiving Break*

Lab: *Thanksgiving Break*

13-3 *Thanksgiving Break*

14-1	*Review*	Text p159/67,69,71;p229/2,3,6,9,14-16, 21,22,25,29,30,31,46.
14-2	*Review*	p232/40;Cspk. p59/1,3,6,9;Cspk. p61/1,3,5.

Lab: Test #3 (without optimization word problems)

14-3 Review of Optimization

15-1 **Test #3,** continued (optimization word problems,4.5)

Notes: You may want to give them four problems and have them work any three or three must work any two.

15-2 *Exam Review*

Notes: You will receive a memo on the rules concerning reviewing for the exam. Basically you can not hand out review material or hold extra review sessions. Your students will have access to all the 25L tests given this semester through the 25L joint Blackboard site. This gives them plenty of practice problems.

Lab: *Teacher's Choice*

Notes: It is up to you if you want to meet with your students to answer questions they have about the tests from ` the other sections. At this point you can only answer specific questions. Decline to answer questions like, "Can you explain what...".

15-3 *Exam Review/ TCE day.*

Notes: (Last day of class business)

If you didn't pass out the Teacher/Course evaluations before, then you must do so today. You should also be sure to take care of the following end-of-semester business:

1. Tell students they may bring one sheet of notes to the final exam. This sheet may be typed, but it must be their own work.

2. Students should have a TI-83 or other approved calculator for the exam. Calculators that have symbolic integration capabilities, like the TI-89, are not allowed.

3. We will attempt to keep the help room operating through the day before the exam. (Some undergraduate workers may have exam week conflicts.)

4. Announce the block exam date and time. Be sure all students understand that the time and date are in the math block time, which is not the same as the time corresponding to the weekly class meeting time.

5. Announce the place of your exam. This information will come from Lewis, and it will be linked from the web page *http://www.math.duke.edu/first_year/* . It does not appear on the Registrar's site, and students have no other source for this information. Emphasize this announcement.

6. You can talk in general terms about the exam. For example, *the exam is written by someone who knows this course well, but is not currently teaching it. The exam will be reviewed by me and all 31L teachers for accuracy and appropriateness. I have not seen the exam yet, but most of these exams are approximately 10 pages long. Our intent is to have an exam that can be completed by most students in about 2.5 hours, but all exams are original and the time required to complete them can vary. Work steadily and be prepared for an exam which could take the full three hours to complete. Be sure to show all of your work in a clear, organized way, because the graders will grant partial credit for partially correct answers. However, these graders will not have time to try to decipher messy, convoluted answers. We have an "army" of graders, and each problem will be graded by the same person on all papers to ensure consistency in grading. The grading scale will be set by the department and will be the same for all sections.*

If students are worried about the block exam and block grading and quotas, etc, you can say something like, *I will turn in semester grades that have approximately the same distribution of letter grades as our class has on the final exam, but that's not a rigid restriction, and I have considerable flexibility. Usually there's not much change, and in the aggregate the exam scores will usually have about the same distribution as test grades for this class have had all semester.*

You can remind students that all sections have the same syllabus, so they should review the topics and problems that appear there. You might also suggest that they use other teachers' tests from this semester as a set of practice problems. They can find these tests posted on *Blackboard.*