Fall 2007 Math 31L Syllabus
Instructor's Version
Textbook: Calculus (4th ed), by Hughes-Hallett, et al.

Day	Section	Topic	Homework

1-1 1.3,1.4 Inverses/Logarithms

Notes: Don't try to cover all of this material in the classroom. It should be a review for students, so you can talk about your policies, etc, and you can leave much of the assignment for the students to read. You'll notice that we have omitted most of the precalculus review in Chapter 1. The thinking on this matter is, if the students don't have the precalculus background for this course, then they should be in Math 25L where there is much more precalculus review.

On this first day, you should briefly describe the course, labs, and your grading policies. You should not make a big issue out of this course being different from the traditional course; simply tell students that it emphasizes applications and why things work. Give out Departmental and individual handouts. Also, tell students about information on the web: they can find math placement information at *www.math.duke.edu/first_year*, as well as links to course home pages, help room schedules, etc. Tell students that they will need to have a TI-83 calculator for this course (or a waiver as described on the web site above).

The lab tomorrow will give the students a good workout with properties of logarithms; thus, it would be good to advise the students to do the homework from this lesson before lab tomorrow, because it will help them with the lab.

Lab: Log plots

Notes: The math in this lab will go much better for students if they have already done the homework from the first lesson. This topic is not in the textbook. One of the purposes of this lab is to get students used to writing a report about mathematics. The two problems at the end of the lab give students a good opportunity to do some writing about the mathematics they have done. You should let them know your expectations clearly.

1-2 2.1 Measuring speed

Notes: This is a good chance to have students work along with you on an exercise. You can ask them to compute some average velocities with their calculators over shrinking intervals of time. Talk about the problem of observing instantaneous velocity and why we need the concept of a limit.

1-3 2.2 Derivative at a point

Notes: Most of your students have had high school calculus. They think they understand these topics, but many don't. Make them compute the derivation at a constant value of x by using the definition.

2-1 1.7, 1.8 Limits

Notes: Do not try to do an ϵ-δ presentation. Use intuition, graphs, and numerical approximations. Tell students that they are NOT responsible for the ϵ-δ definition and examples in the textbook.

Lab: Strategies for Buying Stock

Notes: The idea of this lab is to reinforce the meaning (and usefulness) of difference quotients (and second difference quotients). This lab replaced an old one on US Population.

Day	Section	Topic	Homework

2-2 2.3 The Derivative Function

Notes: Try to convince the students that the derivative is a function and that it means something. To students who took high school calculus it appears to be only a “formula.”

2-3 3.9 Linear Approximations

Notes: This is a very important lesson. You will introduce some of the ideas that will be needed in the Euler's Method lab next week (and which in second semester will be recognized as a first-order Taylor approximation). Draw pictures and try to help students reach an understanding of the ideas.

3-1 2.4 Interpretations of the Derivative

Notes: Lead the class through a careful discussion of some examples. The students are not used to thinking like this—especially those who recite the phrase “instantaneous rate of change.” Example 2 in section 2.4 is a good example of the type of interpretations you should be presenting. Draw some pictures and tie these interpretations to the linear approximations from last time (with $\Delta t = 1$).

Lab: Introduction to Euler's Method

Notes: This topic is new even to the many students who have had a lot of high school calculus. It's here because it's a nice application of the derivative, and using and understanding Euler's method requires students to understand the derivative and basic ideas about slopes of lines. After the lab is over, you can make the connection between this application and the linear approximations. Furthermore, this method will, later in the course, give us an excellent way of arguing (not a formal proof, of course) the validity of the Fundamental Theorem of Calculus.

3-2 2.5 Second Derivative

Notes: Recall that students have encountered the idea of second difference quotients in the lab on Stock Prices. Most will know the concavity interpretation from high school, but be careful in your explanations, because they won't understand why the “rules” they've memorized are true. They are also unlikely to have seen an interpretation of the second derivative other than in the context of acceleration of a moving body or concavity of a curve.

3-3 2.6 Differentiability

Notes: Here's where you can bring up some of the problem cases—but don't use ϵ-δ arguments. Draw pictures and use intuition and do some calculations with the calculator. A good calculator-based demonstration is to have students graph $y = |x|$, and then zoom in on the origin several times. Some students will think their calculators are not zooming in correctly!

4-1 3.1 Power Functions & Polynomials

Notes: This lesson is short and easy. Look at the homework problems and spend some time working an example or two like the hardest ones.

Lab: First & Second Derivatives and Roots

Notes: Rolle's Theorem and the Max/Min Value Theorem should be “discovered” in this lab. Most of this lab is easy for students. You will need to reinforce the observation that Rolle's Theorem applies to functions in general, and not just to polynomials; thus, students cannot simply think in terms of the degree of a polynomial when they try to understand Rolle's Theorem.

Day	Section	Topic	Homework

4-2 3.2 The Exponential Function

Notes: This lesson is an excellent opportunity to have students "discover" something with their calculators. They can produce results like those on page 118 (don't let them see the book at this point). After you establish that the derivative of b^t is (some #) $\cdot\, b^t$, you can have different groups of students compute an approximation for the constant multiplier for different values of b. Then, after you've found a number, which we call e, such that $\frac{d\,e^t}{dt} = e^t$, then you can "borrow" the future result, $\frac{d\,e^{kt}}{dt} = ke^{kt}$, to show that $k =$ (some #) $= ln(b)$. The chain rule lesson, from which we're borrowing a result, comes in section 3.4.

4-3 3.3 Product and Quotient Rules.

Notes: Easy stuff. Have the students work an example so they can see that it cannot be true that $(f \cdot g)'$ is the same as $f' \cdot g'$. The book has a slick way of deriving the quotient rule. Be sure to warn students to start doing the review problems (day 5-1 HW) this weekend because the first major test is coming up next week.

5-1 *Review*

Lab: Test #1

Note: If you're new to Duke, be sure you look at some experienced instructors' old tests before you write your test. A number of copies of old exams are in the resource room (118 Physics) and on line. It would be preferable for you to proctor your own test, but if you can't be in lab, then your lab assistants will have to administer the test; however, you—not your lab assistants— should grade the test.

5-2 3.4 Chain Rule

5-3 1.5 Trig review

Notes: We have not allocated an entire day for a trig review in recent years. But many students are weak in trig, and this is a chance to help them review trig before we start looking at derivatives of trig functions in the next lesson.

6-1 3.5 Der. of Trig Functions

Notes: Students will have had more trouble than you might expect with some of the trig homework problems. Be sure you get to class early and put a few solutions on the board. This lesson gives you another opportunity to have the students make a "discovery" with their calculators. As you consider the derivative of $sin(x)$, point out the difficulty in computing the theoretical limit of the difference quotient. Then have the students set up, in their calculator's graphing menu, a function, such as $\frac{sin(x+.001)-sin(x)}{0.001}$, that can be used to approximate the derivative function. (You can help them come up with this idea by asking, "If we don't know the derivative of a function, how could we set up a function which would approximate it?") This exercise reminds them of the meaning of a difference quotient, and when they graph it, many are amazed that it produces (almost) the graph of the cosine. Once they realize they're getting something like the cosine graph, then you can have them superimpose the graph of $cos(x)$ over the graph of their difference quotient. Near the end of class, you can do the usual "proof," although you will have to rely on numerical approximations of a couple of limits.

Lab: Newton's Law of Motion: An Introduction to Differential Equations

Notes: This lab is the first time students encounter the phrase "differential equations." It's easy for most students, but the main point is to get them used to the terminology and the basic idea of solving an initial value problem.

Day	Section	Topic	Homework

6-2 *Course Pk* Differential Equations.

Notes: This lesson is packed with material, so we have spread it over two days. The strategy here is to cover DEs of the types $\frac{dy}{dt} = f(t)$ and $\frac{dy}{dt} = ky$ today. Then on Friday you can do a quick review, take questions from homework, and the teach students how to work problems of the form $\frac{dy}{dt} = ay + b$. Note that they are taught "z substitution" in this lesson, and then they'll learn separation of variables later in the semestser. **It is of utmost importance that students enter the lab in week 8 with the ability to solve a DE of the form** $\frac{dy}{dt} = ay + b$.

6-3 *Course Pk* Differential Equations

Notes: See the plan outlined for day 6-2.

7-1 *Fall Break*

Lab does not meet this week.

7-2 3.6 Chain Rule and Inverse Functions.

7-3 3.7 Implicit Functions

Notes: We cover implicit differentiation now as an application of the chain rule and for the purpose of preparing students for solving related rates problems in the next lesson.

8-1 4.6 Related Rates

Notes: Warn the students that tomorrow's lab is challenging; tell them that even though it will be hard to recognize in the context of the lab, they will be solving DEs of the form $\frac{dy}{dt} = ay$ and of the form $\frac{dy}{dt} = ay + b$. If you have time, you could review briefly the method they learned for solving the second DE.

Lab: Chemical Rate Equations

Notes: Most students view this lab as the hardest one we do this semester. Be sure you're familiar with the lab, and be sure you find out from your lab assistants exactly what they covered in lab. In the following class be prepared to give an overview of the lab and to help your students finish the lab. You can pick up some lab solutions from Lewis which can be useful to hand out in class on Friday. Your lab assistants will be instructed in their weekly lab prep meeting to leave the last part of the lab for the teacher to discuss in the classroom on Wednesday.

8-2 Chem lab completion

Notes: Be sure you are familiar with the lab. The lab instructors will have been instructed to leave the last part of this lab for you to work through with the students in class. There are a few handouts which you can use in class. Your lab instructors will have been given copies of these handouts, or you can get them from the resource room or from Lewis Blake.

8-3 4.1 Using f' and f''

Notes: You'll be surprised at how many students can recite rules such as, "When the derivative is positive, then the function is increasing," but who have trouble making connections between features of the graphs of f, f', and f''. That lack of understanding is why we do the lab we do in week 4. This lesson is largely a review of the results of that lab. It should help to prepare students for solving the max/min problems which appear in the next lesson.

Day	Section	Topic	Homework

9-1 4.3 Optimization

Notes: The last lesson should have prepared students for this lesson and the next one. You should work very carefully two or three max/min problems of increasing difficulty. Note that using the calculator opens up the possibility of our solving optimization problems which lead to functions with messy derivatives that don't factor nicely.

Tell the students to review their differentiation results for the gateway test tomorrow. Tell them to relax: they can take this test as many times between now and the end of the semester as they need to pass it. (To pass this test, a student a student must work all but one of the problems perfectly.)

Lab: Gateway Test

Notes: Your lab assistant should go to lab armed with several versions of the test. We have plenty of these on file. Students can take make-ups in the help room anytime a Math 31L teacher is on duty. (<u>Note that you must go to the help room prepared with copies of gateway tests after this date</u>.) You can try to motivate students to take care of this business early by pointing out that all who have passed by Thanksgiving week will not have to go to lab that week. If you want to attend this lab, you could use some of the lab time to help students with max/min problems.

9-2 4.4, 4.5 More Optimization; econ example.

9-3 4.7 L'Hôpital's Rule

Notes: The book doesn't prove the most general theorem, but it gives a nice intuitive argument for the case it does cover. You can state the more general version without proof after you've given the argument for the case $x \to a$. And if you like to tell stories, there is a good one about L'Hôpital, Bernoulli, the first calculus textbook, and this theorem. Be sure to remind students to do the review problems this weekend.

10-1 *Review*

Lab: Test #2

10-2 5.1 Distance from Velocity

Notes: You can give the students a handout with some velocity data. Have them use the data to compute a table of approximate distances. Discuss whether they think they have overestimated or underestimated the distance. Discuss how to improve the estimate. Connect this idea to areas of rectangles. Try to convince the students that there should be a limiting value as they reduce the size of the time intervals. Next week's lab is designed to reinforce these ideas by having the students use the calculator to show that the sums approach a limit as the size of the intervals shrinks. Note that The Fundamental Theorem of Calculus will be covered right before the lab next week.

10-3 5.2 The Definite Integral

Notes: This is the definition. The Fundamental Theorem of Calculus is in the next lesson. You can use geometry and the area interpretation to deduce the values of some of the definite integrals. Many students will already have the misconception that the Fundamental Theorem is the definition. You will likely find it a challenge to undo that misconception.

11-1 5.3 Fundamental Theorem of Calculus.

Notes: You'll have to work hard to convince the high school calculus veterans that this is a significant result rather than "just a formula." You can use velocity and distance to motivate the result, and then you have a golden opportunity to connect different parts of the course: you can use the Euler's method construction to start with $F(a)$ and compute an Euler's approximation of $F(b)$ using Euler steps of $F'(t_k)\Delta t$. As $\Delta t \to 0$, with a little hand waving, we get the Fundamental Theorem. Emphasize that the definite integral computes a sum, and that $F(a)$ plus that sum gives us $F(b)$. Many high school calculus veterans will understand for the first time what's really going on.

The part of textbook section 5.3 on “averaging” will be covered in the next lesson.

Lab: Riemann Sums

Notes: Students compute some sums of areas of large numbers of rectangles. The idea is to reinforce the definition of the definite integral and let students see that the sums will approach a limit as the width of the intervals shrinks.

11-2 5.3 Lab review; averaging

Notes: Before you cover averages, it's good to summarize the ideas from the lab. This lesson is another case where some students may have memorized in a previous course the definition you will present, but most students don't understand it. Use a concrete example (such as averaging temperature over a day) to show students that computing a weighted average leads us to the well-known definition of average value of a function. Note that the part of section 5.4 on properties of integrals will be covered in the next lesson rather than today.

11-3 5.4 Properties of Integrals.

Notes: Draw some pictures and write down some difference quotients to show students why these properties are true.

12-1 6.2,6.3 Antiderivatives, Diff. Eqns.

Notes: Most of this was done in the DE lessons in the Coursepack. It should be easy for students by now.

Lab: Varying Density

Note: We usually do only part I of this lab (which was created by Jim Tomberg for Math 26L). It helps to reinforce the meaning of Riemann Sums, and it gives students some practice in using Riemann Sums to help understand how to set up the correct integral.

12-2 6.4 FTC II

Notes: We used to cover this theorem in a lab, but it works better for the teacher to draw some good pictures to explain why the theorem is true. Students should be able to look at your graph and tell you what area is represented by $\int_a^{x+h} f(t)\,dt - \int_a^{x} f(t)\,dt$, which is, of course, the numerator of the difference quotient in the definition of the derivative of the function $\int_a^x f(t)\,dt$.

12-3 11.2 Slopefields

Notes: After you explain the basic idea, you can give the students a new system of DEs and give them a computer generated grid, and ask the students to form groups and work out the slopefield. See Lewis for ideas. The students enjoy this exercise and it works well.

13-1 11.3 Euler's Method

Next: We revisit this idea for three reasons: (1) to reinforce the concept in students' minds; (2) to give students a more general problem, i.e., y' can now be a function of both y and t; and (3) students need to understand this material to do the last part of the lab next week. It works well to divide the class into about three groups, and have each group work the same problem (after a quick review of Euler's method). Choose a DE whose Euler's solution is sensitive to the initial condition and give each group a different initial point. You can ask Lewis for an example.

Lab: Gateway makeup.

Note: Students who have already passed the gateway test should not attend this lab. For students who haven't passed the gateway, attending this lab is mandatory. Warn students that they must have passed the gateway before the final exam (or better, before classes end) or you'll have to turn in an "Incomplete." The end is near.

13-2 *Thanksgiving break*

13-3 *Thanksgiving break*

14-1 11.4 Separation of Variables

Notes: A good way to try to convey understanding is to present this by first doing, then undoing, a Chain Rule differentiation problem. Don't show them the separation of the differentials until you think they understand what's happening. If this plan doesn't make sense, then ask Lewis about it. Students will use separation of variables in lab tomorrow. In fact <u>you may work in today's class the separation of variables part of the IVPs from tomorrow's lab if you wish</u>.

Lab: Net Worth of a Company

Notes: There is a computer generated graph (which carries the solution out farther than the lab requires) which can make a good handout and lead to a good discussion. I give this handout to students <u>after</u> they have generated their own graph as required in the lab manual. If you want to use this handout, be sure you coordinate your activities with your lab assistant, who will have been given a copy in the weekly lab "prep" meeting. (You can find also the handout in the "Lab Materials" drawer of the resource room.)

14-2 11.6 Applications and Modeling

14-3 11.7 Population models

15-1 Review

Notes: If you'd prefer to give out the Teacher/Course evaluation forms today rather than on the last day, then you are encouraged to do so. That way, students are more likely to respond to the semester as a whole rather than making comments about Test #3 only, and you'll have plenty of time to talk about grades, exam procedures, etc, on the last day.

Lab: Test #3

15-2 Teacher's option

15-3 Last day of class business

Notes: If you didn't pass out the Teacher/Course evaluations last Monday or at some other time, then you must do so today. You should also be sure to take care of the following end-of-semester business:

1. Tell students they may bring one sheet of notes to the final exam. This sheet may be typed, but it must be their own work.

2. Students should have a TI-83 or other approved calculator for the exam. Calculators that have symbolic integration capabilities, like the TI-89, are not allowed.

3. We will attempt to keep the help room operating through the day before the exam. (Some

undergraduate workers may have exam week conflicts.)

4. Announce the block exam date and time. Be sure all students understand that the time and date are in the math block time, which is not the same as the time corresponding to the weekly class meeting time.

5. Announce the place of your exam. This information will come from Lewis, and it will be linked from the web page *http://www.math.duke.edu/first_year/* . It does not appear on the Registrar's site, and students have no other source for this information. Emphasize this announcement.

6. You can talk in general terms about the exam. For example, *the exam is written by someone who knows this course well, but is not currently teaching it. The exam will be reviewed by me and all 31L teachers for accuracy and appropriateness. I have not seen the exam yet, but most of these exams are approximately 10 pages long. Our intent is to have an exam that can be completed by most students in about 2.5 hours, but all exams are original and the time required to complete them can vary. Work steadily and be prepared for an exam which could take the full three hours to complete. Be sure to show all of your work in a clear, organized way, because the graders will grant partial credit for partially correct answers. However, these graders will not have time to try to decipher messy, convoluted answers. We have an "army" of graders, and each problem will be graded by the same person on all papers to ensure consistency in grading. The grading scale will be set by the department and will be the same for all sections.*

If students are worried about the block exam and block grading and quotas, etc, you can say something like, *I will turn in semester grades that have approximately the same distribution of letter grades as our class has on the final exam, but that's not a rigid restriction, and I have considerable flexibility. Usually there's not much change, and in the aggregate the exam scores will usually have about the same distribution as test grades for this class have had all semester.*

You can remind students that all sections have the same syllabus, so they should review the topics and problems that appear there. You might also suggest that they use other teachers' tests from this semester as a set of practice problems. They can find these tests posted on *Blackboard.*