Spring 2004 Math 31L Syllabus
Instructor's Version
Textbook: Calculus (3rd ed), by Hughes-Hallett, et al.

General comments on the first week: Students will think they know this material, and you will bore them by trying to lecture on all of the topics in the first chapter. We recommend that you lecture on selected topics only, and spend some time having them work in groups on challenging problems based on the material they are reading.

Day	Section	Topic	Homework	*Revised December 20, 2003*

1-2 1.1 Functions and Change 1.1/4,10,14,21,23,28,30,32,33,34,38,39; *Calculator Drill* Part 1.

Notes: Don't try to cover this material in the classroom. It should be an easy review for students, so you can talk about your policies, etc, and leave the assignment for the students to read. Briefly describe the course, labs, and grading policies. You should not make a big issue out of this course being different from the traditional course; simply tell students that it emphasizes applications and why things work. Give out Departmental and individual handouts. Also, tell students about information on the web: they can find math placement information at *www.math.duke.edu/first_year*, as well as links to course home pages, help room schedules, etc. Tell students that they will need to have a TI-83 calculator for this course (or a waiver as described on the web site above).

You should notice that this homework assignment and the next couple include parts of the calculator drill. I suggest you tell students that you won't collect that part, and that that part of the assignment is on the syllabus just to be sure they can handle the necessary features of their calculators before they attempt to do the labs.

Lab: Risk Factors for Breast Cancer

Notes: This lab is mathematically easy. The students fit a line to data (not with least-squares, but only by visual estimation) and then interpret the slope and intercept in context. The main purpose is to get students accustomed to interpreting math in context and to give them practice in writing a math report. You should let them know your expectations clearly.

1-3 1.2 Exponential Functions 1.2/9-11,14,17,24-27,30,32-34,36,38; *Calculator Drill* Part 2.

Notes: A good classroom exercise is to give the students a table of data (which is an exponential function of time—but don't tell them) and ask them to compute ratios of consecutive values. Then show them what that means. The population of Mexico taken from the book works fine.

2-1 1.3 New Functions from Old 1.3/1,5,6,9,11,12,15,24-29,31,34,36; *Calculator Drill* Part 3.

Notes: Many students do not understand composition of functions, even though they think they do. Give a short lecture, including inverse functions, and then have students work together in class on some hard problems such as 28-30 at the end of the section. You may be surprised at how hard many students find those problems.

Day	Section	Topic	Homework
2-2	1.4	Logarithmic Functions	1.4/4,5,8,18,23,24,26,30,34,36,37,42,45,49,50.

Notes: The change of base theorem for logs is not covered in the textbook. It's useful later in the course if you show them that result now: $log_b(x) = log_B(x)/log_B(b)$. In lab tomorrow students will make use of some of the other properties of logs when they construct log-log and semilog plots. You can emphasize that they will be making a connection between data and theory.

Lab: Log plots

Notes: In section 1.2 students used ratios to uncover an underlying exponential relationship. In this lab they learn a better method, one which does not require equal changes in the independent variable. You can make a comment to students about what they'll be doing. This topic is not in the textbook.

2-3	2.1	Measuring speed	2.1/2,4,5,7-9,12-16,18-20.

Notes: This is a good chance to have students work along with you on an exercise. You can ask them to compute some average velocities with their calculators over shrinking intervals of time. Talk about the problem of observing instantaneous velocity and why we need the concept of a limit.

3-1 *Martin Luther King, Jr., holiday*

3-2	2.2	Limits	2.2/1,13-16,20,22,30-32,34-38.

Notes: Do not try to do an ϵ-δ presentation. Use intuition, graphs, and numerical approximations. Tell students that they are NOT responsible for the ϵ-δ definition and examples in the textbook.

Lab: Strategies for Buying Stock

Notes: This is a new lab. The idea is to reinforce the meaning (and usefulness) of difference quotients (and second difference quotients). This lab replaces an old one on US Population.

3-3	2.3	Derivative at a point	2.3/2,5-7,9-11,14,16,17,26,29.

Notes: Most of your students have had high school calculus. They think they understand these topics, but many don't. Make them compute the derivation at a constant value of x by using the definition.

4-1	2.4	The Derivative Function	2.4/1-3,5,8,13,14,21,28-30,32,35,37.

Notes: Try to convince the students that the derivative is a function and that it means something. To students who took high school calculus it appears to be only a “formula.”

4-2	3.9	Linear Approximations	3.9/3,9,14 [Replace first sentence with: *Assume f is concave up*], 15 [Replace first sentence with: *Assume f is concave down*].

Notes: This is a very important lesson. You will introduce some of the ideas that will be needed in the Euler's Method lab tomorrow (and which in second semester will be recognized as a first-order Taylor approximation). Draw pictures and try to help students reach an understanding of the ideas. *Advise students that they should do at least some of the homework problems before lab so they'll understand the lab better.*

Day	Section	Topic	Homework

Lab: Introduction to Euler's Method

Notes: This topic is new to almost all students, even those who have had a lot of high school calculus. It's here because it's a nice application of the derivative, and using and understanding Euler's method requires students to understand the derivative and basic ideas about slopes of lines. After the lab is over, you can make the connection between this application and the linear approximations. Furthermore, this method will, later in the course, give us an excellent way of arguing (not a formal proof, of course) the validity of the Fundamental Theorem of Calculus.

4-3 2.5 Interpretations of the Derivative 2.5/1-3,5,10,14,16,18;
begin day 5-2 review problems.

Notes: Lead the class through a careful discussion of some examples. The students are not used to thinking like this—especially those who recite the phrase "instantaneous rate of change." Example 2 on page 87 is a good example of the type of interpretations you should be presenting. Draw some pictures and tie these interpretations to the linear approximations from last time (with $\Delta t = 1$).

5-1 2.6 Second Derivative 2.6/1-3,8-14,17,18,21-23;
complete day 5-2 review problems.

Notes: Recall that students have encountered this idea in the lab on Stock Prices. Most will know the concavity interpretation from high school, but be careful in your explanations, because they won't understand why the "rules" they've memorized are true. They are also unlikely to have seen an interpretation of the second derivative other than in the context of acceleration of a moving body or concavity of a curve.

5-2 *Review* p49/10,17-20,22,26-28,34,38,40;
p99/2,3,8,12-14,16,18,22-25,27-30,36,39.

Lab: Test #1

Note: If you're new to Duke, be sure you look at some experienced instructors' old tests before you write your test. A number of copies of old exams are in the resource room (118 Physics). It would be preferable for you to proctor your own test, but if you can't be in lab, then your lab assistants will have to administer the test; however, you, not your lab assistants, should grade the test.

5-3 2.7 Cont. and Differentiability 2.7/1-3,6-12,14,16.

Notes: Here's where you can bring up some of the problem cases—but don't use ϵ-δ arguments. Draw pictures and use intuition and do some calculations with the calculator.

6-1 3.1 Power Functions & Polynomials 3.1/1,2,4,6-8,10-15,22,24,31,32,45,
48-51,54,55,57,59,60.

Notes: This lesson is short and easy. Look at the homework problems and spend some time working an example or two like the hardest ones.

Day	Section	Topic	Homework

6-2 3.2 The Exponential Function 3.2/1,4,10,12,17,22,36,38,40,41,43-45,47.

Notes: This lesson is an excellent opportunity to have students "discover" something with their calculators. They can produce results like those on page 113 (don't let them see the book at this point). After you establish that the derivative of b^t is (some #) $\cdot\, b^t$, you can have different groups of students compute an approximation for the constant multiplier for different values of b. Then, after you've established e such that $\frac{d\,e^t}{dt} = e^t$, then you can "borrow" the future result, $\frac{d\,e^{kt}}{dt} = ke^{kt}$, to show that $k = (\text{some \#}) = ln(b)$. The chain rule lesson, from which we're borrowing a result, comes two lessons hence.

Lab: Newton's Law of Motion: An Introduction to Differential Equations

Notes: This lab is the first time students encounter the phrase "differential equations." It's easy for most students, but the main point is to get them used to the terminology and the basic idea of solving an initial value problem.

6-3 3.3 Product and Quotient Rules 3.3/1,2,4,5,11,15,16,19,20,28,31,32, 34-37,41,44,46,48,49,52.

Notes: Easy stuff. Have the students work an example so they can see that it cannot be true that $(f \cdot g)'$ is the same as $f' \cdot g'$. The book has a slick way of deriving the quotient rule.

7-1 3.4 Chain Rule 3.4/1,4,6,9,15,17,27,36,45,49,51,52,54, 55,57,58,61,63.

7-2 *Course Pk* Differential Equations Course Pack p75/1a,c,e,2a,b,c,3a,c,4a,c,5,10a,b,13.

Notes: This lesson is <u>packed</u> with material, and students will feel overwhelmed by the time they get to their homework problems. The strategy here is to cover all of this material quickly today. (I tell my students to "fasten your seat belts.") Then in lab tomorrow the students can work together with guidance from the lab instructors on the homework problems. <u>It is of utmost importance that students enter next week's lab with the ability to solve a DE of the form</u> $\frac{dy}{dt} = ay + b$.

Lab: Work differential equations problems from day 7-2 homework.

7-3 *Course Pk* Differential Equations Course Pack p75/3b,d,4b,9,10c,20,21;
Textbook section 1.5: trig review 1.5/13,18,19,31,36,38,42.

Notes: Give a brief summary of the techniques covered last time and work some homework problems. If you have time, you can give students a brief trig review. (Check the trig homework problems to see what they are expected to be able to do.) You can leave the trig review up to students to read if you prefer.

8-1 3.5 Derivatives of Trig Functions 3.5/4,5,9,11,14,16,18,25,27,31,41-43,47,49,50.

Notes: Students will have more trouble than you might expect with some of the trig homework problems. Be sure you get to class early and put a few solutions on the board. This lesson gives you another opportunity to have the students make a "discovery" with their calculators. As you consider the derivative of $sin(x)$, point out the difficulty in computing the theoretical limit of the difference quotient. Then have the students set up, in their calculator's graphing menu, a function, such as $\frac{sin(x+.001)-sin(x)}{0.001}$, that can be used to approximate the derivative function. (You can help them "rediscover" this idea by

asking, “If we don't know the derivative of a function, how could we set up a function which would approximate it?”) This exercise reminds them of the meaning of a difference quotient, and when they graph it, many are amazed that it produces (almost) the graph of the cosine. Once they realize they're getting something like the cosine graph, then you can have them superimpose the graph of $cos(x)$ over the graph of their difference quotient. Near the end of class, you can do the usual “proof,” although you will have to rely on numerical approximations of a couple of limits.

8-2 3.6 Applications of Chain Rule 3.6/1,2,6,7,10,13,14,17,18,20,25,37,38,40, 42-45,53,54.

Notes: You'll see that there are some related rates problems in the homework, even though we haven't used the phrase, “related rates.” Just work those problems as a simple application of the chain rule. On day 9-2 we give students an old-fashioned related rates presentation. Warn the students that tomorrow's lab is challenging; tell them that even though it will be hard to recognize in the context of the lab, they will be solving DEs of the form $\frac{dy}{dt} = ay$ and of the form $\frac{dy}{dt} = ay + b$.

Lab: Chemical Rate Equations

Notes: Most students view this lab as the hardest one we do this semester. Be sure you're familiar with the lab, and be sure you find out from your lab assistant exactly what they covered in lab. In the following class be prepared to give an overview of the lab and to help them finish the lab. You can pick up some lab solutions from Lewis which can be useful to handout in class on Friday. You lab assistants will be instructed in their weekly lab prep meeting to leave the last part of the lab for the teacher to discuss in the classroom.

8-3 Lab review and completion .

Notes: Spend all the time that is necessary to complete the lab. You may want to have the students sit in their lab groups as you go over parts of the lab. If you want to get some more information and handouts on this lab, check the “Lab Materials” drawer in the resource room.

9-1 3.7 Implicit Functions; Rel. Rates 3.7/1,2,11,12,20,21,26,30.

Notes: We want implicit differentiation and solving related rates problems to appear simply as applications of the chain rule.

9-2 Related Rates Course Pack p79/1,3-6

Lab: First & Second Derivatives and Roots

Notes: Rolle's Theorem and the Max/Min Value Theorem should be “discovered” in this lab. Most of this lab is easy for students, and it's a “breather” after the Chem Rates Lab.

9-3 4.1 Using f' and f'' 4.1/10,11,15,19,26,27,29,30,35,40,42; begin day 11-2 review problems.

Notes: You'll be surprised at how many students can recite rules such as, “When the derivative is positive, then the function is increasing,” but who have trouble making connections between features of the graphs of f, f', and f''. That lack of understanding is why we do the lab we do this week. This lesson and yesterday's lab should prepare them for solving the max/min word problems which appear after test #2. You should go ahead

and define the terms “global maximum” and “global minimum”, as well as “local max/min,” even though the textbook does not do so until section 4.3. Some of the review problems for this block of material require that students find global extrema. Optimization word problems will not be covered until after the upcoming test, so related rates word problems and max/min word problems don't come too close together.

10 *Spring Break*

11-1 3.10 L'Hôpital's Rule 3.10/1-3,5,11-19.

Notes: The book doesn't prove the most general theorem, but it gives a nice intuitive argument for the case it does cover. You can state the more general version without proof after you've given the argument for the case $x \to a$. And if you like to tell stories, there is a good one about L'Hôpital, Bernoulli, the first calculus textbook, and this theorem.

11-2 *Review* p99/13; p158/6,8; p159/54,60-62,70-72,74-77,83a; p212/7,10,12,24,38; Course Pack p76/4d,10c,15.

Lab: Test #2

11-3 4.3 Optimization 4.3/3,4,7,12,19,21,23,24,26,31,33; gateway practice on page 159.

Notes: The lab and lessons just before spring break should have prepared students for this lesson and the next two. You should work very carefully two or three max/min word problems of increasing difficulty. Note that using the calculator opens up the possibility of our solving optimization problems which lead to functions with messy derivatives that don't factor nicely. Remind students that the gateway test is coming up next week.

12-1 4.4 Optimization (economics) 4.4/1,3-7,9,10,14.

Notes: Spend sufficient time on homework problems to be sure students understand how to do all of the assigned problems. Then work some economics max/min problems. The quick derivation of the economics result that profit is maximized when Marginal Revenue = Marginal Cost makes a good introduction. Check the homework problems to get some other ideas.

12-2 4.5 More Optimization 4.5/2,5-7,12,18,21,23; 4.4/15.

Notes: Be sure your lab assistant has copies of gateway tests for lab tomorrow. We will probably have several versions to hand out to the lab assistants in their weekly “lab prep” meeting, but you should check to make sure all is in order. If you'd prefer, you may write your own or make minor modifications in old ones, copies of which can be found in the resource room. Tell the students to review their differentiation results for the gateway test tomorrow and to relax: they can take this test as many times between now and the end of the semester as they need to pass it. (See the Instructor's Manual for more details.)

Lab: Gateway Test

Notes: Your lab assistant should go to lab armed with several versions of the test. Students can take make-ups in the help room anytime a Math 31L teacher is on duty. (Note that you must go to the help room prepared with copies of gateway tests after this date.)

Day	Section	Topic	Homework

12-3 5.1 Distance from Velocity 5.1/1,3-7,9,11,12,14.

Notes: You can give the students a handout with some velocity data. Have them use the data to compute a table of approximate distances. Discuss whether they think they have overestimated or underestimated the distance. Discuss how to improve the estimate. Connect this idea to areas of rectangles. Try to convince the students that there should be a limiting value as they reduce the size of the time intervals. Next week's lab is designed to reinforce these ideas by having the students use the calculator to show that the sums approach a limit as the size of the intervals shrinks. Note that The Fundamental Theorem of Calculus will be covered at the end of the week after the lab.

13-1 5.2 The Definite Integral *Calculator Drill* Part 5; 5.2/2,3,6,28-30,32,36,37.

Notes: This is the definition. The Fundamental Theorem of Calculus comes at the end of the week. You can use geometry and the area interpretation to deduce the values of some of the definite integrals. Many students will already have the misconception that the Fundamental Theorem is the definition. You will likely find it a challenge to undo that misconception.

13-2 5.3 Averaging 5.3/1,4-7,10-14,18,20,21,26,29-31.

Notes: This is another case where some students may have memorized in a previous course the definition of average value. Use a concrete example (such as averaging temperature over a day) to show students that computing a weighted average leads us to the well-know definition of average value of a function.

Lab: Riemann Sums

Notes: Students compute some sums of areas of large numbers of rectangles. The idea is to reinforce the definition of the definite integral and let students see that the sums will approach a limit as the width of the intervals shrinks.

13-3 5.4 Fundamental Theorem of Calculus; properties of the definite integral

. Homework: 5.4/1,2,4-20,22; 5.3/19,22,23.

Notes: You'll have to work hard to convince the high school calculus veterans that this is a significant result rather than “just a formula.” You can use velocity and distance to motivate the result, and then you have a golden opportunity to connect different parts of the course: you can use the Euler's method construction to start with $F(a)$ and compute an Euler's approximation of $F(b)$ using Euler steps of $F'(t_k)\Delta t$. As $\Delta t \to 0$, with a little hand waving, we get the Fundamental Theorem. Emphasize that the definite integral computes a sum, and that $F(a)$ plus that sum gives us $F(b)$. Many high school calculus veterans will understand for the first time what's really going on.

If you were to cover all the properties of definite integrals carefully, there would be enough material for two lessons here. (We spread this material over two lessons in the fall, but there wasn't enough time in the spring syllabus, given the constraints of labs, etc.) You can pick one or two properties and draw a picture and write down a difference quotient to show students why the property is true. Then tell students to read about the other properties in the textbook. At the beginning of the next class, you can briefly review the properties which you don't get to today.

Day	Section	Topic	Homework

14-1 6.2,6.3 Antiderivatives, Diff. Eqns. 6.2/2,4,6,8,10-12,15,16,18,19,22,28,35,41,55, 56,58,59,70-72,75,81,82; 6.3/11,12,16,17,21-23.

Notes: Most of this was done in the DE lessons in the course pack. It should be easy now, so you don't have to cover all of this again. The main new thing to introduce is the notation and terminology for indefinite integrals.

14-2 6.4 FTC II 6.4/1-3,5,7,8,10,11,14,16-22.

Notes: We used to cover this theorem in a lab, but it works better for the teacher to draw some good pictures to explain why the theorem is true. Students should be able to look at your graph and tell you what area is represented by $\int_a^{x+h} f(t)\,dt - \int_a^x f(t)\,dt$, which is, of course, the numerator of the difference quotient in the definition of the derivative of the function $\int_a^x f(t)\,dt$.

Lab: Gateway makeup.

Note: Students who have already passed the gateway test should not attend this lab. For students who haven't passed the gateway, attending this lab is mandatory. Warn students that they must have passed the gateway before the final exam (or better, before classes end) or you'll have to turn in an "Incomplete." The end is near.

14-3 11.3 Euler's Method 11.3/3a,b,4a,5a,9; begin day 15-3 review problems.

Next: We revisit this idea for three reasons: (1) to reinforce the concept in students' minds; (2) to give students a more general problem, i.e., y' can now be a function of both y and t; and (3) students need to understand this material to do the last part of the lab next week. It works well to divide the class into about three groups, and have each group work the same problem (after a quick review of Euler's method). Choose a DE whose Euler's solution is sensitive to the initial condition and give each group a different initial point. You can ask Lewis for an example.

15-1 11.4 Separation of Variables 11.4/1,6,9,11,15,18,21,22,25,34; 11.5/6,18,19,23.

Notes: A good way to try to convey understanding is to present this by first doing, then undoing, a Chain Rule differentiation problem. Don't show them the separation of the differentials until you think they understand what's happening. If this doesn't make sense, then ask Lewis about it.

HW note: Because we have not covered u-substitution, students will have to work problem #25 using "guess and check." You may want to warn them about this problem.

15-2 11.2 Slopefields 11.2/3,4,6,8-10; 11.5/4; 11.3/2,6.

Notes: After you explain the basic idea, you can give the students a new system of DEs and give them a computer generated grid, and ask the students to form groups and work out the slopefield. See Lewis for ideas. The students enjoy this exercise and it works well.

Lab: Net Worth of a Company

Notes: There is a computer generated graph (which carries the solution out farther than the lab requires) which can make a good handout and lead to a good discussion. I give this handout to students after they have generated their own graph as required in the lab

manual. If you want to use this handout, be sure you coordinate your activities with your lab assistant, who will have been given a copy in the weekly lab "prep" meeting. (You can find the handout in the "Lab Materials" drawer of the resource room.)

Because there is a major test coming up on Monday, there is probably not time for either a quiz or a report on this lab. You can promise students that, in lieu of a report or lab quiz, on the upcoming test there will be a question or two based on this lab.

15-3 Review p212/27,30,31; p252/1,3,4,6,9,10,14,16,18,22-24,27, 29,31,35,36; p285/32,43,45,47,51-53; p552/3,10,18,30,31,34,36,37.

Notes: If you'd prefer to give out the Teacher/Course evaluation forms today rather than on the last day, then you are encouraged to do so. That way, students are more likely to respond to the semester as a whole rather than making comments about Test #3 only, and you'll have plenty of time to talk about grades, exam procedures, etc, on the last day.

16-1 Test #3

16-2 *TCE Day*

Notes: If you didn't pass out the Teacher/Course evaluations last Friday or Monday, then you must do so today.