Fall 2008 Math 32L Syllabus (Instructor Version)

Textbook: Calculus (4th ed), by Hughes-Hallett, et al.

Day Reference Topic *Revised August 5, 2009*

1-1 Intro. Selected topics from Math 31L

Note: You will have to use about half of this period doing the beginning of semester business, such as giving students an overview of the course, your grading plans, the help room, calculator rules, homework procedures, etc. For the second half of the period we have a handout with exercises based on the main topics from Math 31L which we assume students know. You can have students work in class in groups on the exercises. They probably won't finish in class, but you can tell them to complete the worksheets individually outside of class.

1-2 Probability #1 in the Course Pack: *Events*

Idea: Probability is introduced early in this course because it provides an interesting context in which to motivate series. It also lays the groundwork for statistics applications, which require students to recall the definition of the derivative, use the FTC II, and apply improper integrals and Taylor series. You should give students a preview of this "thread" which runs through much of the semester.

Lab: Air Pollution: Fine Particulate Matter

Idea: Students have to use a simple Riemann sum, such as a left-hand sum, to approximate the value of an integral. You may be surprised how many students will have trouble putting together theoretical results with a table of data. You can make some comments to "connect" this lab with the ideas reviewed on the first day of class.

1-3 Probability #2 in the Course Pack: *Random Variables*

In Class: This lesson and the next one used to be covered in one day, but we found that the students had a lot of trouble understanding the material and getting the homework problems done. Be sure to allow time to take questions from the last homework, and after you carefully present this lesson on random variables, work as many examples as you have time. You may want to restructure this lesson and the next one so that you cover easy examples with both random variables expected values (currently scheduled for the following lesson) in the first lesson; then you would cover harder examples of random variables and expected values on the second day. If you do this, you'll need to adjust the homework assignments for these two days. If you stick with the original plan of covering expected values on the next class day, then at the end of class you should raise the question of what we would **expect** to happen in the long run–a question to be answered next time.

2-1 Probability #3 in the Course Pack: *Expected Values*

In Class: This material has been put into a separate lesson so you will have time to answer questions from the last homework, to present the motivation for the definition of expected value, and to work several nontrivial examples of expected value. An excellent segue into the next lesson is to conclude today's lesson with the following thought experiment: *toss a coin until you get heads. Let the r.v. X denote the number of tosses it takes. List the sample space. Look at probabilities of each outcome. Raise the question: is the sum of those probabilities 1?*

2-2 9.2 Geometric Series

In Class: Be aware that examples of geometric series from probability and the essence of the geometric series theorem will be done in lab tomorrow. You should be sure you know exactly what's in that lab before you write your notes for this class. In the next lesson in the classroom you will introduce the general concept of partial sums, so this is a chance to ease into those ideas in the restricted context of geometric series. Use the phrase "partial sum" of the geometric series when you show the derivation of the theorem to lay the groundwork for the next lesson. Be sure you work some word problems and some problems with sums that don't "start at 0."

Lab: *Probability and Geometric Series*

Idea: The strategy here is to introduce series as they arise naturally in a concrete setting. We do not formally define series and sums of series until day 2-3.

2-3 Series#1 in the Course Pack; section 9.1 in the textbook: *Partial Sums*

In Class: Geometric series and the geometric series theorem have been covered; however, the students have seen no general presentation on series. The purpose of this lesson is to introduce partial sums in a formal way, and you can refer to the results from the lab as an example of the more general definitions and results. This material was written for the course pack (a.k.a., Lab Manual) because the textbook has only scant coverage of partial sums and almost no exercises which reinforce the concept of partial sums. Students will have some homework problems where they must use simple partial fractions (which method has not been covered) to produce a telescoping sum. Try to do one of these in class, but you don't have to give a presentation of partial sums in general–rather, just enough to get this job done. The last part of this section of the textbook, the n-th term test, will not be covered until day 4-2.

3-1 7.1 Integration by Substitution

In Class: It's a good idea to review the Fundamental Theorem of Calculus before you begin this lesson.

Note: One of the reasons for doing several lessons on integration at this time is to let the series material "perk" in the students minds for a few days before we cover more topics on series. Indeed, one of the pedagogical strategies of this course is to introduce series very early, and then to spread the lessons on series out over the entire semester so students have plenty of time to absorb the concepts and not much time to forget about series. It is helpful to explain this strategy to students at the beginning of this lesson so they will understand why we're switching topics.

3-2 7.2 Integration by Parts

In Class: You may want to mention to students that one way to think about this procedure is "undoing" the product rule.

Lab: Integrating to Infinity

Note: This lab was first used in the spring of 2002. We dropped an old lab on World Population, which we used to do in the first lab meeting. This lab introduces the idea of computing integrals with an infinite upper bound, so students will have this tool available when you cover the integral test soon. Later in the semester there is an in-class lesson in which we will cover improper integrals in general, including other types of improper integrals. For now, all the students need to know is how to integrate to infinity. It is good to tell students that the ideas in this lab will be the foundation of an important lesson on series next week.

3-3 7.4 Alg. Identities (including partial fractions, but **not** including trig substitution)

In Class: Students have encountered a simple case of partial fractions in working with partial sums of series. You should work some slightly harder problems here, but you do not have to cover the most general case. On the gateway test we will expect students only to be able to do a problem in which the denominator has two linear factors.

Trig substitution is not formally included in this course; however, we expose them to the idea by assigning one or two problems where they are given an explicit (trig) substitution to try on a very easy problem. Later in the course if you use some old gateway test, you'll have to be on the lookout for a trig substitution problem which might be on an old gateway test. You should also point out to students that even though trig substitution appears in this section of the textbook we will not include it in this course.

4-1 7.5,7.6 Approximating Definite Integrals

In Class: If you prefer, you can motivate Simpson's Rule by giving the parabolic arc description (but don't try to give a formal derivation of the formula) instead of the weighted averaging method that is done in the book. There is a handout available if you'd like to use it.

4-2 Series#1 in the Course Pack: *n-th Term Test*

9.3 (through example #1)

In Class: Ask if students have any questions from the lab, *Integrating to Infinity*. After you've done that thoroughly, then cover the n-th term test. We use only the first part of section 9.3 in the textbook, because that's where the n-th Term Test is, and the second half includes the integral test which we cover next time using material in the course pack. One of the homework problems is a proof of the n-th Term Test, and it can work well to have students do that problem in groups at the end of class today.

Lab: Normal Data Sets, Part 1; Series Worksheet, Part 1 (p 219 *Course Pack*)

Note: The students will work the first part of the Normal Data Lab, in which they choose what data they will be gathering over the next few weeks. The data will be used in the lab in week 8. After the Normal Data business is done, have students work in teams on assigned series problems. This practice with series is here, because students will have begun to have lots of questions about series and convergence issues.

4-3 Series #3 in the Course Pack: *Integral Test*

In Class: We use material in the course pack, rather than the textbook, because we want to include coverage on error bounds. Students learned how to integrate to infinity in the week 3 lab; they will learn about other improper integrals on day 5-3.

HW note: In the homework the students will have to find an integral of xe^{-x^2} by guess-and-check, since we have not covered *u*-substitution.

5-1 9.4 Comparison Test; Absolute Convergence Theorem, Limit Comparison Test.

In Class: Cover the two Comparison Tests and the Absolute Convergence Theorem. You will see that this section of the textbook contains enough material for at least three lessons. We will cover the Ratio Test, which is also in section 9.4, just before we cover power series later in the course. Also, we will discuss the Alternating Series Theorem when we need it to treat the endpoints of an interval of convergence later in the course.

When you talk about the Absolute Convergence Theorem, you'll have to do some hand-waving and talk about how cancellation of positive and negative terms could make a series more likely to converge. Then later, when we do cover the Alternating Series Theorem, you can "recall" these remarks as a lead-in to that theorem.

5-2 Review

Lab: Test #1

5-3 7.7, 7.8 *Improper Integrals (definitions and basics)*

In Class: Remember that students have done a lab (in week 3) about integrating to infinity. You can remind them of what they have already seen and introduce the term "improper integral"; then, cover the other cases of improper integrals.

Important Homework Note: Students should work all problems in the "Background" section of the lab, *Present Value and Future Value* (on pages 109-110 of the *Course Pack*) before lab next week. It is important that students complete this precalculus material before lab, so they will have most of their lab time available to think about the applications in the lab which require calculus. You could enforce this requirement, if you think it's necessary, by collecting their lab background work on the day before lab or at the beginning of the lab.

6-1 8.1 *Areas and Volumes*

In Class: Present volumes by slicing. The textbook doesn't cover volumes of revolution until section 8.2, but it may be easier to go ahead and cover volumes by disks and washers today, as well. We will cover arc length in the next lesson. In any case look over this lesson and the next one before you write your lesson plans, so you can decide exactly how much you want to cover each day. We do not cover the shell method for finding volumes. *In this lesson and in the next lesson you should emphasize the process of setting up the correct integral by referring to Riemann sums, and you should discourage students from memorizing formulas*.

6-2 8.2 Arc length; more volumes

In Class: Present the method for computing arc length, as well as covering any solids of revolution which you didn't cover on day 6-1.

Lab: Present Value and Future Value

Note: Students should have completed the "Background" part before the lab meets.

6-3 8.7 *Distribution Functions*

Note: This begins the treatment of <u>continuous</u> probability and statistics.

Lab preparation: Tell students that they should have all of the data (which they've been collecting since week 4) stored in one calculator by Thursday of next week. It is also helpful if they come to lab with a printout of their sorted data.

7-1 *Fall Break*

7-2 8.8 *Probability; Distributions*

Lab: Varying Density, part III; gateway practice

Note: Students did the first two parts of this lab in Math 26L or Math 31L. Thus, we're using only part III here to give students some practice in constructing Riemann sums in a context other than volume and area. And students won't be ready for the *Normal Data Lab* until next week.

7-3 8.8 *Normal Distributions*

8-1 10.1 *Taylor Polynomials*

In Class: After you present the construction of Taylor polynomials, you can have the students graph a couple of examples on their calculators. You should talk about accuracy, and you should raise the issue that if we try to use an "infinite polynomial," we face convergence questions. Explain to students that we will soon use our convergence theorems to answer those convergence questions. Some students will miss this class because of the upcoming fall break, but that's one reason we have a second lesson on Taylor polynomials on Wednesday after fall break.

8-2 10.1 *Taylor Polynomials*

In Class: Review the basic theory. Give thorough answers to all questions which students have on the homework problems.

Lab: Normal Data Sets, Part 2

Note: One of the steps in this lab requires students to approximate values of a cumulative distribution function by looking at their sorted data which they should have gathered over the last two weeks. This experience will be the first time that many of them come to an understanding of what the distribution function is. The lab also requires students to approximate values of the density function by setting up difference quotients with the values they computed for their distribution function. This process reinforces their basic understanding of the derivative. Note that time is set aside on day 9-1 for students to make presentations on their work. **An alternative way to handle day 9-1 is shown in the handout at the end of this syllabus**.

8-3 9.4 Ratio Test

In Class: Because we have already covered the Absolute Convergence Theorem, you can present the full version of the Ratio Test. Your presentation in class should show the dependence of the Ratio Test upon the Geometric Series Theorem, although we do not expect you to give a formal proof of the Ratio Test. We take some problems from the course pack, because there aren't very many good ones in the textbook.

Note: The gateway test will be given in lab in week 9, and there are practice problems in this homework assignment. (Please refer to the Instructor's Manual for information on gateway policy.) You should tell students that they do not have to hand in all of the practice integration problems. A good way for students to practice is to read all of the problems and decide which method should be used; then they can check the answer key, which indicates the best method. Students should, of course, work some of the problems completely. 9-3 Series #4 in the Course Pack: *Alternating Series*

In Class: The coverage of this theorem in section 9.3 is all right, but we use the version in the course pack because it has more good homework problems. Note that the last part of *Series #4*, which is on the Extended Ratio Test, has already been covered.

9-1 *Student Presentations*

In Class: Each lab team should make a 5-minute presentation on their normal lab project. **Or refer to the handout at the end of this document for an alternative way to handle this day.**

HW Note: The homework assignment consists of series review problems.

9-2 Series #4 in the Course Pack: *Alternating Series*

In Class: The coverage of this theorem in section 9.4 of the textbook is all right, but we use the version in the course pack because it has more good homework problems. Note that the last part of *Series #4*, which is on the Extended Ratio Test, has already been covered.

Note: Remind students that they will have a gateway test in lab tomorrow. You should also remind them of the rules (which you can check in the Instructor's Manual for this course).

Lab: Gateway Test

9-3 9.5 *Convergence of Power Series*

In Class: Tell students that we're looking at these issues of convergence so we can extend Taylor polynomials to Taylor series. Explain intervals of convergence and how to handle the endpoint questions.

10-1 10.2 *Taylor Series, including binomial series*

In Class: You can now prove that the Taylor polynomials for e^x and $sin\, x$ converge for all x. (You may choose to raise the question, “to what,” but we will not answer that formally–rather we'll use graphical evidence. Be sure you also work a problem, such as $ln(x)$ at base 1, which has a small interval of convergence. Also, derive the binomial series (or if you don't have time, tell students to read the derivation given in the textbook). Tell students that they should be able to recognize the series for e^x, $sin\, x, cos\, x$, and the binomial series.

10-2 10.3 *Using Taylor Series*

In Class: We spend two class days on this section in the textbook. After devoting significant time to reviewing homework problems, show students how to derive a new series from known series by substitution, differentiation, and multiplication. Getting new series by integrating known series can wait until the next lesson.

Lab: Series Solutions of Initial Value Problems

Note: This is a revision of an old project. This method of solving DEs is not in the textbook. One purpose of the lab is to let students see another application of series.

10-3 10.3 *Using Taylor Series*

In Class: Spend a lot of time answering students' questions on homework (and lab, if necessary). Show students how to create a new series by integration. You may want to show them two ways to deal with the constant term: by using an IVP, and by using a definite integral. It makes an interesting classroom example to show students how to derive the result $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$

11-1 Catch-up day
11-2 Review
Lab: Test #2
11-3 10.5 *Fourier Series with period 2π*

In Class: Your coverage in this lesson should go through the middle of page 464 in the textbook. Be sure to point out that, even though we derive the formulas with integrals over $[-\pi, \pi]$, we can use any one complete period that we choose. (They need to know this to do some of the homework problems efficiently.) On day 12-1 we will cover periodic functions with periods other than 2π. You should tell students that once they learn how to handle the case for a period of 2π, the adjustment to a more general period is easy (i.e., if you don't read the confusing explanation in the book!). We do not cover the part of this section on the Energy Theorem.

12-1 10.5 *General Fourier Series* ***DANGER***

In Class: Regarding the general case of Fourier series, the presentation in the book is very confusing. We recommend that you outline a derivation of the formulas for the Fourier coefficients for a period of $2p$, where p is a positive number representing half a period. It is easy to show that $a_k = \frac{1}{p}\int_{-p}^{p} f(t)\, cos(\frac{\pi}{p}kt)\, dt$, etc, by using a method identical to the one used to derive the formulas for the case of period 2π (thus, it is not necessary to work all the details again). **Advise students NOT to read the material in the book on how to handle the general period**.

Homework: Some of the answers in the solutions manual are wrong.

12-2 10.5 *Fourier Review*

In Class: Students will want to see most of the homework from general Fourier series done. You can talk about checking symmetry. Have them graph the Fourier polynomials on their calculators.

Lab: Fourier Analysis of Musical Sound
12-3 11.8 *Predator-prey with phase plane*

In Class: A good strategy here is to explain the model, explain how to construct a phase plane, and carefully sketch a few slopes in the slope field. Then after you're sure the students have the idea, give them a complete computer-generated phase plane. (You can find one in the resource room or get one from Lewis.) You can ask students to pick a few different starting points and trace a trajectory from each point. Then you can raise questions such as, what would happen if the population of the prey were suddenly increased (say by park rangers or worm lovers or ...)?

13-1 11.8 *SIR Model with phase plane*

In Class: A strategy similar to that used in the last class, where you give students a computer-generated phase plane–after they sketch a few slopes themselves, will work well here. (Again, check the resource room or ask Lewis.) This is a great opportunity to ask students some questions they can estimate from the phase plane: for example, what is the maximum number of people infected at the same time? What is the total number of people who caught the disease during the time it ran its course through the population? And the question which really impresses students: how many people would have to be inoculated to head off an epidemic? They will be impressed at being able to estimate this information from the phase plane. Tell the students that in the upcoming lab they will see how to make a modification in the DEs to reflect the possibility of "recovered" people becoming susceptible again.

13-2 11.9 *Phase Plane Analysis (more general cases; nullclines; equilibria)*

Lab: Limited Immunity in Epidemics, Part 1 (SIRS, a variation on the SIR model)

Note: We don't cover Part 2 simply because the lab is long enough with Part 1 only.

13-3 Completion of lab

14-1 11.10 *Oscillations*

In Class: Present the DE $\frac{d^2y}{dt^2} = -ky$, and find the general solution by explaining why it must represent oscillating motion, by using our knowledge of derivatives, and doing some guesswork. Show why a solution of the form, $y = Ae^{rt}$ (which would be reasonable to consider, in light of the similarity to the DE $\frac{dy}{dt} = ky$) cannot work because of the negative sign—unless we use complex numbers.

We have chosen not to pursue the solution with complex numbers, but you could comment to students that that is a possibility which works; i.e., we can use complex numbers to solve a real problem to get a real solution. They've never imagined such a tactic, but unfortunately we don't have time to do the details.

14-2 *Thanksgiving break*

Lab: *Thanksgiving*

14-3 *Thanksgiving break*

15-1 *TCE; review*

In Class: Have students fill out Teacher/Course evaluations. Use the rest of the time to review. [TCEs could be done on Wednesday, if you'd rather.]

15-2 *Review*

Lab: Test #3

15-3 *Last day of class business*

Notes: If you didn't pass out the Teacher/Course evaluations last Monday or Wednesday, then you must do so today. You should also be sure to take care of the following end-of-semester business (shown on the next page):

Last Day of Class Business

1. Tell students they may bring one sheet of notes to the final exam. This sheet may be typed, but it must be their own work.

2. Students should have a TI-83 or other approved calculator for the exam. Calculators that have symbolic integration capabilities, like the TI-89, are not allowed.

3. We will attempt to keep the help room operating through the day before the exam. (Some undergraduate workers may have exam week conflicts.)

4. Announce the block exam date and time. Be sure all students understand that the time and date are in the <u>math block time</u>, which is <u>not</u> the same as the time corresponding to the weekly class meeting time.

5. Announce the place of your exam. This information will come from Lewis, and it will be linked from the web page *http://www.math.duke.edu/first_year/* . It does <u>not</u> appear on the Registrar's site, and students have no other source for this information. Emphasize this announcement.

6. You can talk in general terms about the exam. For example, *the exam is written by someone who knows this course well, but is not currently teaching it. The exam will be reviewed by me and all 32L teachers for accuracy and appropriateness. I have not seen the exam yet, but most of these exams are approximately 10 pages long. Our intent is to have an exam that can be completed by most students in about 2.5 hours, but all exams are original and the time required to complete them can vary. Work steadily and be prepared for an exam which could take the full three hours to complete. Be sure to show all of your work in a clear, organized way, because the graders will grant partial credit for partially correct answers. However, these graders will not have time to try to decipher messy, convoluted answers. We have an "army" of graders, and each problem will be graded by the same person on all papers to ensure consistency in grading. The grading scale will be set by the department and will be the same for all sections.*

If students are worried about the block exam and block grading and quotas, etc, you can say something like, *I will turn in semester grades that have approximately the same distribution of letter grades as our class has on the final exam, but that's not a rigid restriction, and I have considerable flexibility. Usually there's not much change, and in the aggregate the exam scores will usually have about the same distribution as test grades for this class have had all semester.*

You can remind students that all sections have the same syllabus, so they should review the topics and problems that appear there. You might also suggest that they use other teachers' tests from this semester as a set of practice problems. They can find these tests posted on *Blackboard.*

[Note to teachers: this handout was given out with the Normal Lab, Part 2. The written reports, student-made handouts, and quiz were all handled on the following Monday, which is day 9-1 on this syllabus.]

Normal Lab Report and Quiz Schedule

Math 32L, Blake, Spring 2005

Report

Your team should turn in one copy of the completed *Normal Data Sets Report Form*, on Monday, March 7.

Handout

Along with your completed report form, your team should provide a one-page handout for the class. This handout should include the following:

- A title or description which shows what data you gathered.
- A picture of your histogram.
- The sample mean and standard deviation of your data.
- A picture of the normal density function graphed over the plot of the density function which you approximated for your data.
- The percentage of your data values which lie within one, and within two, standard deviations of the mean.
- A brief statement summarizing your conclusion in the last part of the lab report.
- Your names.

Quiz

After the report and handout are collected on Monday, March 7, there will be an individual, closed-notes quiz based on this lab.