Spring 2009 Math 32L Syllabus (Instructor Version)
Textbook: Calculus (4th ed), by Hughes-Hallett, et al.

Day Reference Topic *Revised August 15, 2008*

1-2 Probability #1 in the Course Pack: *Events*

Idea: Probability is introduced early in this course because it provides an interesting context in which to motivate series. It also lays the groundwork for statistics applications, which require students to recall the definition of the derivative, use the FTC II, and apply improper integrals and Taylor series. You should give students a preview of this "thread" which runs through much of the semester. Before you make your plans for this class, you should be sure to read the notes for the next class day.

1-3 Probability #2 in the Course Pack: *Random Variables*

In Class: This lesson and the next one used to be covered in one day, but we found that the students had a lot of trouble understanding the material and getting the homework problems done. Be sure to allow time to take questions from the last homework, and after you carefully present this lesson on random variables, work as many examples as you have time. At the end of class you should raise the question of what we would **expect** to happen in the long run–a question to be answered next time.

2-1 Probability #3 in the Course Pack: *Expected Values*

In Class: This material has been put into a separate lesson so you will have time to answer questions from the last homework, to present the motivation for the definition of expected value, and to work several nontrivial examples of expected value. An excellent segue into the next lesson is to conclude today's lesson with the following thought experiment: *toss a coin until you get heads. Let the r.v. X denote the number of tosses it takes. List the sample space. Look at probabilities of each outcome. Raise the question: is the sum of those probabilities 1?*

Lab: *Probability and Geometric Series*

Idea: The strategy here is to introduce series as they arise naturally in a concrete setting. We do not formally define series and sums of series until day 2-2.

2-2 9.2 Geometric Series

In Class: Be aware that geometric series and the essence of the geometric series theorem were introduced in the lab yesterday. You should be sure you know exactly what's in that lab. This lesson gives you a chance to summarize the results of the lab and to review briefly the derivation of the Geometric Series Theorem. In the next lesson you will introduce the general concept of partial sums, so this is a chance to ease into those ideas in the restricted context of geometric series. Use the phrase "partial sum" of the geometric series when you show the derivation of the theorem to lay the groundwork for tomorrow's lesson. Be sure you work some word problems and some problems with sums that don't "start at 0."

2-3 Series#1 in the Course Pack; section 9.2 in the textbook: *Partial Sums*

In Class: Geometric series and the geometric series theorem have been covered; however, the students have seen no general presentation on series. The purpose of this lesson is to introduce partial sums in a formal way, and you can refer to the results from the lab as an example of the more general definitions and results. This material was written for the course pack (a.k.a., Lab Manual) because the textbook has only scant coverage of partial sums and almost no exercises which reinforce the concept of partial sums. Students will have some homework problems where they must use simple partial fractions (which method has <u>not</u> been covered) to produce a telescoping sum. Try to do one of these in class, but you don't have to give a presentation of partial fractions in general–rather, just enough to get this job done. The last part of this section, the n-th term test, will not be covered until the next lesson.

3-1 *Martin Luther King, Jr. holiday*

Lab: Integrating to Infinity

Note: This lab introduces the idea of computing integrals with an infinite upper bound, so students will have this tool available when you cover the integral test on Friday. There is an in-class lesson on day 6-3 in which we will cover improper integrals in general, including other types of improper integrals. For now, all the students need to know is how to integrate to infinity.

3-2 Series#1 in the Course Pack: *n-th Term Test and sum of series*

9.3 (through example #1)

In Class: Use much of the class time to go over homework from partial sums and to deal with conceptual questions which the students may have about series. After you've done that thoroughly, then cover the n-th term test. We use only the first part of section 9.3 in the textbook, because that's where the n-th Term Test is, and the second half includes the integral test which we cover next time using material in the course pack. Homework problem #28 is a proof of the n-th Term Test, and it may work well to have students do that problem in groups at the end of class today.

3-3 Series #3 in the Course Pack: *Integral Test*

In Class: Ask if students have any questions from the lab, *Integrating to Infinity* before you start this lesson. We use material in the course pack, rather than the textbook, because we want to include coverage on error bounds. Students learned how to integrate to infinity in the last lab; they will learn about other improper integrals on day 6-3.

HW note: In the homework the students will have to find an integral of xe^{-x^2} by guess-and-check, since we have not covered *u*-substitution.

4-1 9.4 Comparison Test; Absolute Convergence Theorem; Limit Comparison Test.

In Class: Cover the Comparison Test and the Absolute Convergence Theorem. You will see that this section of the textbook contains enough material for at least three lessons. We will cover the Ratio Test, which is also in section 9.4, <u>next time</u>. Also, we will discuss the Alternating Series Theorem on day 4-3. When you talk about the Absolute Convergence Theorem, you'll have to do some hand-waving and talk about how cancellation of positive and negative terms could make a series more likely to converge. Then on day 4-3, you can "recall" these remarks as a lead-in to the Alternating Series Theorem.

Lab: Normal Data Sets, Part 1; Series Worksheet, Part 1 (p 213 *Course Pack*)

Note: The students will work the first part of the Normal Data Lab, in which they choose what data they will be gathering over the next two weeks. The data will be used in the lab in week 8. Last year was the first time we made the series worksheet part of this lab, and we had mixed results. We're looking for a new way to deal with this exercise.

4-2 9.4 Ratio Test

In Class: Because we have already covered the Absolute Convergence Theorem, you can present the full version of the Ratio Test. Your presentation in class should show the dependence of the Ratio Test upon the Geometric Series Theorem, although we do not expect you to give a formal proof of the Ratio Test. We take some problems from the course pack, because there aren't very many good ones in the textbook.

4-3 Series #4 in the Course Pack: *Alternating Series*

In Class: The coverage of this theorem in section 9.4 of the textbook is all right, but we use the version in the course pack because it has more good homework problems. Note that the last part of *Series #4*, which is on the Extended Ratio Test, has already been covered.

5-1 Review

Lab: Test #1

5-2 7.1 *Integration by Substitution*

In Class: It's a good idea to review the Fundamental Theorem of Calculus before you begin this lesson.

5-3 7.5,7.6 *Approximating Definite Integrals*

In Class: If you prefer, you can motivate Simpson's Rule by giving the parabolic arc description (but don't try to give a formal derivation of the formula) instead of the weighted averaging method that is done in the book. This lesson is scheduled for this day because the upcoming lab requires students to approximate integrals numerically.

6-1 7.2 *Integration by Parts*

Lab: Air Pollution: Fine Particulate Matter

Idea: Students have to use a simple Riemann sum, such as a left-hand sum, to approximate the value of an integral. You may be surprised how many students will have trouble putting together theoretical results with a table of data.

6-2 7.4 Algebraic identities

In Class: Students have encountered a simple case of partial fractions in working with partial sums of series. You should work some slightly harder problems here, but you do not have to cover the most general case. On the gateway test we will expect students only to be able to do a problem in which the denominator has two linear factors.

6-3 7.7, 7.8 *Improper Integrals (definitions and basics)*

In Class: Remember that students have done a lab (in week 3) abut integrating to infinity. You can remind them of what they have already seen and introduce the term “improper integral”; then, cover the other cases of improper integrals.

Note: The gateway test will be given in lab on Tuesday of next week, and there are practice problems in the homework assignment. (Please refer to the Instructor's Manual for information on gateway policy.) You should tell students that they do not have to hand in all of the practice integration problems. A good way for students to practice is to read all of the problems and decide which method should be used; then they can check the answer key, which indicates the best method. Students should, of course, work some of the problems completely.

7-1 8.7 *Distribution Functions*

Note: This begins the treatment of continuous probability and statistics. Also, be sure your lab assistant has copies of a gateway test for lab tomorrow. (The tests will be put in your mail box.) If you'd prefer, you may write your own or make minor modifications in old ones, copies of which can be found in the resource room.

HW Note: The homework assignment includes some review of series convergence theorems.

Lab: Gateway Test

7-2 8.8 *Probability; Distributions*

7-3 8.8 *Normal Distributions*

Lab preparation: Tell students that they should have all of their data in one calculator (or in a lap top computer) by Tuesday, so they will be ready to do the calculations required in lab next week. It will also be very helpful if they have their data sorted and printed, so they can use the printout in lab. Also, be sure you read through the next week-and-a-half of lessons and labs so you can decide ahead of time exactly how you plan to handle the grading of the upcoming lab. You'll need to tell both your students and your lab assistants on Monday what you plan to do.

HW Note: The homework assignment includes some review of series convergence theorems.

8-1 8.1 *Areas and Volumes*

In Class: Present volumes by slicing. The textbook doesn't cover volumes of revolution until section 8.2, but it may be easier to go ahead and cover volumes by disks and washers today, as well. We will cover arc length in the next lesson. In any case look over this lesson and the next one before you write your lesson plans, so you can decide exactly how much you want to cover each day. We do not cover the shell method for finding volumes. *In this lesson and in the next lesson you should emphasize the process of setting up the correct integral by referring to Riemann sums, and you should discourage students from memorizing formulas*.

Lab: Normal Data Sets, Part 2

Note: One of the steps in this lab requires students to approximate values of a cumulative distribution function by looking at their sorted data which they should have gathered over the last two weeks. This experience will be the first time that many of them come to an understanding of what the distribution function is. The lab also requires students to approximate values of the density function by setting up difference quotients with the values they computed for their distribution function. This process reinforces their basic understanding of the derivative. Friday's class will be partially used to having students prepare their results for presentations on Monday.

8-2 8.2 Arc length; more volumes

In Class: Present the method for computing arc length, as well as covering any solids of revolution which you didn't cover on day 8-1. Note that the next lesson can be a “catch-up” day.

8-3 *Catch up day; prepare for presentation.*

In Class: We have found it challenging to keep up with lab-related questions and homework questions this week; thus, we have built in this day for catching up. To the extent that you have time, you can let students work on their upcoming presentations (or handouts or quiz) for the Normal Distribution Lab. You should give students some guidelines for what you expect them to do in the next class. (A handout Lewis used in the spring of '05 is attached to the end of this document.)

Note: There is a lot of material in the lab next week, although it's not a particularly difficult lab. It is important that students complete the precalculus “Background” material before Tuesday, so they will have most of their lab time available to think about the applications in the lab which require calculus. You should emphasize that students must have this background part of the lab completed by Tuesday. You could enforce this requirement, if you think it's necessary, by collecting their lab background work on Monday or at the beginning of the lab.

9-1 *Presentations*

In Class: Each lab team should make a 5-minute presentation and/or provide a handout to the class on their normal lab project. Please see the notes on day 8-3 for more comments relevant to this lesson.

HW Note: There are some series review problems in the homework assignment.

Lab: Present Value and Future Value

Note: Students should have completed the “Background” part before the lab meets.

9-2 10.1 *Taylor Polynomials with base point 0*

In Class: After you present the construction of Taylor polynomials (with base point 0), you can have the students graph a couple of examples on their calculators. You should talk about accuracy, and you should raise the issue that if we try to use an “infinite polynomial,” we face convergence questions. Explain to students that we will soon use our convergence theorems to answer those convergence questions. We will cover how to handle base points other than 0 in the next lesson. Also, point out to students that there will be a major test on Tuesday following their week of spring break.

9-3 10.1 *Taylor Polynomials with arbitrary base point*

In Class: Review the basic theory, and then show students how to construct a Taylor Polynomial at an arbitrary base point. Give thorough answers to all questions which students have on the homework problems. Many students will miss this class because of the upcoming spring break.

10 *Spring Break*

11-1 Review

Lab: Test #2

11-2 9.5 *Convergence of Power Series*

In Class: Tell students that we're looking at these issues of convergence so we can extend Taylor polynomials to Taylor series. Explain intervals of convergence and how to handle the endpoint questions.

11-3 10.2 *Taylor Series, including binomial series*

In Class: You can now prove that the Taylor series for e^x and $sin\,x$ converge for all x. (You may choose to raise the question, "to what," but we will not answer that formally–rather we'll use graphical evidence. Be sure you also work a problem, such as $ln(x)$ at base 1, which has a small interval of convergence. Also, derive the binomial series (or tell students to be sure to read about the binomial series in the textbook). Tell students that they should be able to recognize the series for e^x, $sin\,x, cos\,x$, and the binomial series.

12-1 10.3 *Using Taylor Series*

In Class: We spend two class days on this section in the textbook. After devoting significant time to reviewing homework problems, show students how to derive a new series from known series by substitution, differentiation, and multiplication. Getting new series by integrating known series can wait until the next lesson.

Lab: Series Solutions of Initial Value Problems

Note: This is a revision of an old project. This method of solving DEs is not in the textbook. One purpose of the lab is to let students see another application of series.

12-2 10.3 *Using Taylor Series*

In Class: Spend a lot of time answering students' questions on homework (and lab, if necessary). Show students how to create a new series by integration. You may want to show them two ways to deal with the constant term: by using an IVP, and by using a definite integral. It makes an interesting classroom example to show students how to derive the result $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$

12-3 10.5 *Fourier Series with period 2π*

In Class: Your coverage in this lesson should go through the middle of page 506 in the textbook. Be sure to point out that, even though we derive the formulas with integrals over $[-\pi, \pi]$, <u>we can use any one complete period that we choose</u>. (They need to know this to do some of the homework problems efficiently.) On day 13-2 we will cover periodic functions with periods other than 2π. You should tell students that once they learn how to handle the case for a period of 2π, the adjustment to a more general period is easy (i.e., if you don't read the confusing explanation in the book!). We do <u>not</u> cover the part of this section on the Energy Theorem.

13-1 10.5 *Fourier series review*

In Class: Go over (or have students put on the board) all of the homework problems. Explain homework #17 carefully, because understanding how to work that problem will help students in the lab tomorrow. In fact if you have time to outline the concepts of the lab, that would be helpful. This lesson has been added because we have seen that students need a review of Fourier series before the lab. Fourier series with more general periods will be covered on Wednesday, but students don't need that for the lab and they need to have these ideas firmly in mind before proceeding to the general case.

Lab: Fourier Analysis of Musical Sound

Note: There is an optional follow-up to this lab. For more information check the description of the lab activities for week 14.

13-2 10.5 *General Fourier Series* ***DANGER***

In Class: Before you present the general case, you may want to give an overview of yesterday's lab. Regarding the general case of Fourier series, the presentation in the book is very confusing. We recommend that you outline a derivation of the formulas for the Fourier coefficients for a period of $2p$, where p is a positive number representing half a period. It is easy to show that $a_k = \frac{1}{p} \int_{-p}^{p} f(t)\, cos(\frac{\pi}{p}kt)\, dt$, etc, by using a method identical to the one used to derive the formulas for the case of period 2π (thus, it is not necessary to work all the details again). **Advise students NOT to read the material in the book on how to handle the general period**.

Also, if you decide to give a quiz on Friday based on the Fourier lab, you'll need to announce that today. You should also read the information about the optional project that could be assigned next week, and decide before you give this lesson what you will do, so you can tell students about your plans. See the week 14 lab for details.

13-3 10.5 *Fourier Review; possible lab quiz*

In Class: Students will want to see most of the last homework problems done. You can talk about checking symmetry. Have them graph the Fourier polynomials on their calculators. If you want to give a quiz on the Fourier lab, you could do so today–but keep it under 30 minutes, so you can review the last homework. If you choose to give a more substantial Fourier quiz, you could do so during the lab next week. (Please see the notes for that lab below.) Lewis recommends giving a Fourier quiz in next week's lab.

14-1 11.10 *Oscillations*

In Class: Present the DE $\frac{d^2y}{dt^2} = -ky$, and find the general solution by explaining why it must represent oscillating motion, by using our knowledge of derivatives, and doing some guesswork. Show why a solution of the form, $y = Ae^{rt}$ (which would be reasonable to consider, in light of the similarity to the DE $\frac{dy}{dt} = ky$) cannot work because of the negative sign—unless we use complex numbers.

We have chosen not to pursue the solution with complex numbers, but you could comment to students that that is a possibility which works; i.e., we can use complex numbers to solve a real problem to get a real solution. They've probably never imagined such a tactic, but unfortunately we don't have time to do the details.

Lab: Fourier Quiz or Gateway Makeup: *Teacher's Choice*.

Note: One way to handle the end of semester testing is to give a Fourier Quiz in lab today. Such a quiz would cover not only the lab, but all Fourier material that we have covered. Then in the last week, you could give a shortened Test #3 on that Monday, because you wouldn't have to include any Fourier questions again. Otherwise, you can simply make this a gateway test make-up day.

14-2 11.8 *Predator-prey with phase plane*

In Class: A good strategy here is to explain the model, explain how to construct a phase plane, and carefully sketch a few slopes in the slope field. Then after you're sure the students have the idea, give them a complete computer-generated phase plane. (You can find one in the resource room or get one from Lewis.) You can ask students to pick a few different starting points and trace a trajectory from each point. Then you can raise questions such as, what would happen if the population of the prey were suddenly increased (say by park rangers or worm lovers or ...)?

14-3 11.8 *SIR Model with phase plane*

In Class: A strategy similar to that used in the last class, where you give students a computer-generated phase plane–after they sketch a few slopes themselves, will work well here. (Again, check the resource room or ask Lewis.) This is a great opportunity to ask students some questions they can estimate from the phase plane: for example, what is the maximum number of people infected at the same time? What is the total number of people who caught the disease during the time it ran its course through the population? And the question which really impresses students: how many people would have to be inoculated to head off an epidemic? They will be impressed at being able to estimate this information from the phase plane.

Tell the students that in the upcoming lab they will see how to make a modification in the DEs to reflect the possibility of "recovered" people becoming susceptible again.

15-1 11.9 *Phase Plane Analysis (more general cases; nullclines; equilibria)*

Lab: Limited Immunity in Epidemics, Part 1 (SIRS, a variation on the SIR model)

Note: We don't cover Part 2 simply because the lab is long enough with Part 1 only.

15-2 Review of lab

In Class: In case some students didn't finish Part 1 of the lab yesterday, you should go over the major results. In particular be sure to review the phase plane and to discuss the questions in (i) of Part 1.

15-3 Review for test

16-1 Test #3

Note: If you prefer, you can give the last test in lab this week. But if you plan to return their graded tests to them before classes end, you'll have to do some fast grading.

Lab: Gateway test makeup

16-2 *TCE Day*

[Note to teachers: this handout was used on day 8-3.]

Normal Lab Report and Quiz Schedule

Math 32L, sections 1 & 2; Blake, Spring 2005

Report

Your team should turn in one copy of the completed *Normal Data Sets Report Form*, on Monday, March 7.

Handout

Along with your completed report form, your team should provide a one-page handout for the class. This handout should include the following:

- A title or description which shows what data you gathered.
- A picture of your histogram.
- The sample mean and standard deviation of your data.
- A picture of the normal density function graphed over the plot of the density function which you approximated for your data.
- The percentage of your data values which lie within one, and within two, standard deviations of the mean.
- A brief statement summarizing your conclusion in the last part of the lab report.
- Your names.

Quiz

After the report and handout are collected on Monday, March 7, there will be an individual, closed-notes quiz based on this lab.