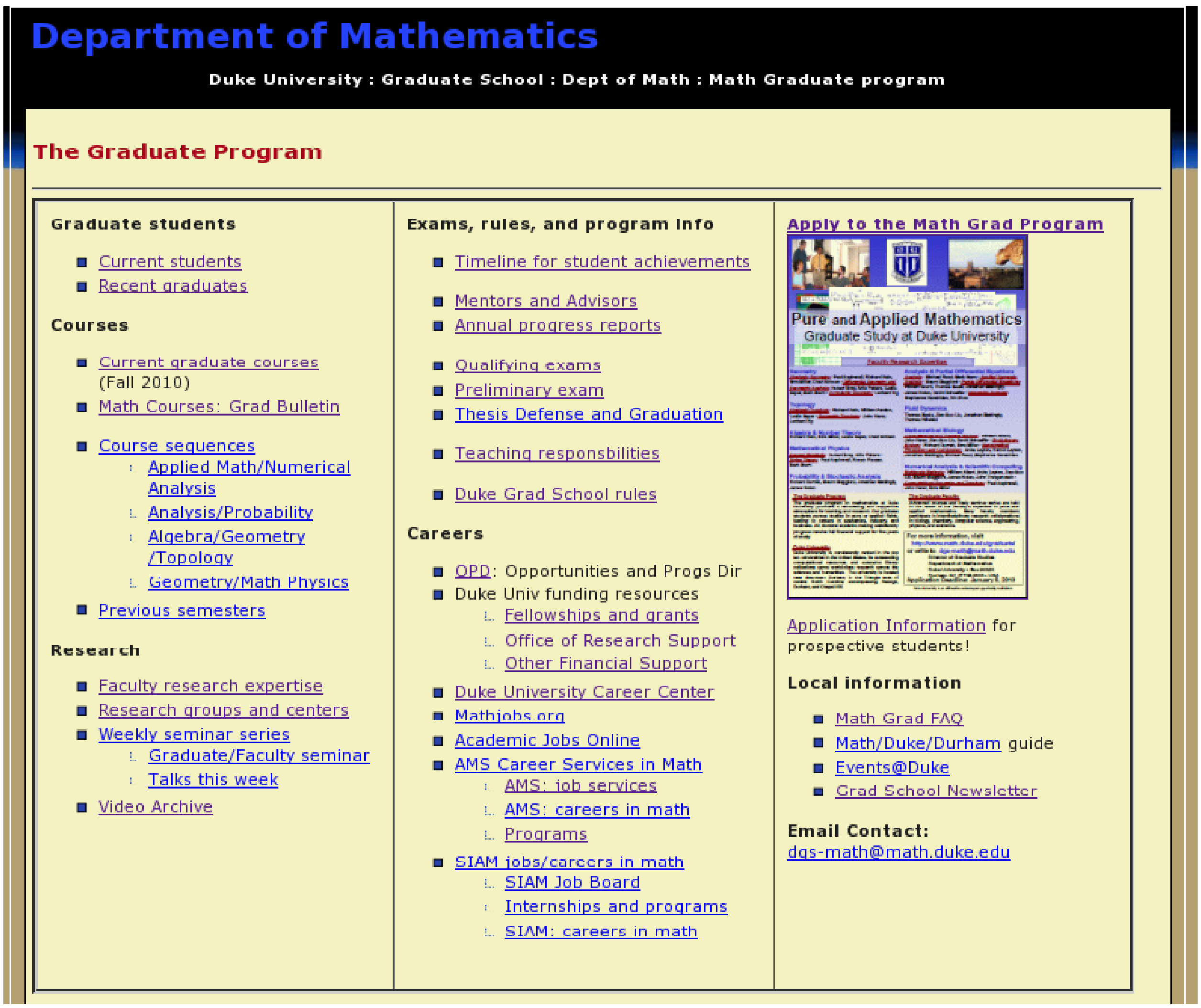
Mathematics Graduate Handbook (2010)



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Application Information for prospective students!

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- Math Grad FAQ
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Email Contact:
dgs-math@math.duke.edu

October 16, 2010

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Preface

This first edition of the Math Grad Handbook (MGH) is just a text-formatted version of several of the more important webpages from the graduate program website. (Other pages with links to sources outside of Duke are not included.) This document may make for easier reference and searching for some bits of information about the program ....

Tom Witelski (October 16, 2010)

(Thesis rules info updated Oct 16,2010)

Chapter 1

Department and Graduate School rules and requirements

1.1 Student timeline: an overview of activities

1. **First Year**

   - International students are provided with the necessary papers to obtain a visa before the beginning of the academic year. For more information, see Duke Visa Services or Duke International House.
   - Before orientation week there is a pre-qualifier program run by senior graduate students. This intensive course culminates in the first written qualifying examinations. The written Qual exams must be completed by the end of the first year. The oral Qual exams must be completed before the end of the fall term of second year.
   - **Orientation week** is the week before classes (see the Duke University calendar for the correct dates).
   - Entering international students are required to take English Language Proficiency Tests (during orientation week)
   - All entering students are required to attend an orientation on Responsible Conduct of Research (RCR) (Natural Sciences/Engineering track, during orientation week). University Ph.D. graduation requirements involve taking **6 more credit-hours of RCR courses!**
   - First-year students will begin teacher training in the fall of their first year. Fall term courses includes the Math 390 teaching seminar course and three graduate level mathematics courses. During spring, students will take four graduate courses, and do two practice teaching lectures. Any changes must be approved by the DGS.

     Teacher training begins on **Monday of orientation week**. All graduate students who will be financially supported by assisting with calculus laboratories or by teaching **must** have attended the training sessions before their duties begin; thus, entering graduate students who will be running calculus labs in the fall or in the spring, must participate in this training during the orientation week.
     The details of teacher training can be found by viewing the outline and the schedule of the most recent teacher training workshop.
   - Students will follow registration policies for fall and spring semester courses.
     Students register for summer semesters ONLY if they are being supported by research grants or departmental research funds for the summer.
   - Students are required to file an annual progress report with the Director of Graduate Studies by April 15.

2. **Second Year**

   - Teacher training begins on **Monday of orientation week.** Any graduate student who has not already attended the training sessions, and who expects to run calculus labs or to teach in the fall or spring, should participate in teacher training during orientation week. The details of teacher training can be found by viewing the outline and the schedule of the most recent teacher training workshop. Graduate students who have already participated in the training and who are scheduled to teach their own class, should check the training week schedule for other meetings required of new teachers or of all teachers in a particular course.
     The details of teacher training can be found by viewing the outline and the schedule of the most recent teacher training workshop.
   - Second year students who are teaching take three courses per semester. Reading courses or other changes require approval from a mentor/advisor and the DGS.
   - Second-year students will teach a course during one semester.
   - If the student has not yet passed the oral qualifying examination, it must be taken and passed before the beginning of the spring semester.
   - Preparation for the preliminary exam should be underway: committing to a research area and picking a research advisor are the first steps!

- Students will follow registration policies for fall and spring semester courses.
  This applies for ALL years, even if you are just registering for ”CTN 1” continuation.
  Students register for summer semesters ONLY if they are being supported by research grants or departmental research funds for the summer.
- Students should continue to take RCR forums to complete the 12 RCR credit hours required for graduation.
- Students are required to file an annual progress report with the Director of Graduate Studies by April 15.

3. **Third year**

- Students will be supported by faculty research grants or teaching assignments.
- The preliminary exam must be passed during the third year.
- After passing the preliminary exam, research and work leading to the Ph.D. thesis are top priority.
- Students are encouraged to take any remaining courses or mini-courses that are relevant to their research direction.
- Students will follow registration policies for fall and spring semester courses.
  This applies for ALL years, even if you are just registering for ”CTN 1” continuation.
  Students register for summer semesters ONLY if they are being supported by research grants or departmental research funds for the summer.
- Students should continue to take RCR forums to complete the 12 RCR credit hours required for graduation.
- Students are required to file an annual progress report with the Director of Graduate Studies by April 15.

4. **Fourth year**

- Students will be supported by faculty research grants or teaching assignments.
- Students are encouraged to take relevant mini-courses.
- Students should be making steady progress toward graduation with their thesis defense during their final year of study (typically the fifth year for students who came straight from undergraduate programs, and in the fourth year for students who came with a master’s degree).
- Students should get detailed feedback on their research progress towards graduation during their pen-ultimate year (usually fourth year).
- Students will follow registration policies for fall and spring semester courses.
  This applies for ALL years, even if you are just registering for ”CTN 1” continuation.
  Students register for summer semesters ONLY if they are being supported by research grants or departmental research funds for the summer.
- Students should continue to take RCR forums to complete the 12 RCR credit hours required for graduation.
- Students are required to file an annual progress report with the Director of Graduate Studies by April 15.

5. **Fifth year**

- Students will be supported by faculty research grants or teaching assignments.**Fifth year is the final year of guaranteed financial support!** Plan ahead accordingly!
  Grad school rules lead to different policies on office space and financial support after the fifth year.
- Job applications for academic positions starting the following year are usually due in mid Fall.

- Students will attend the Departmental Grant Writing Workshop in the Fall to help prepare their CV and job applications.
- See thesis defense for details on University rules regarding the thesis and the final defense!
- Students will follow registration policies for fall and spring semester courses.
  This applies for ALL years, even if you are just registering for "CTN 1" continuation.

- Students should continue to take RCR forums to complete the 12 RCR credit hours required for graduation.
- Students are required to file an annual progress report with the Director of Graduate Studies by April 15.

1.2 Qualifying exams

The qualifying exam is part of the process used by the Mathematics Department to satisfy the Graduate School requirements for the Master of Arts degree. This examination consists of two components:

1. Written exams in basic analysis and linear algebra
2. Oral exams on two graduate-level topics

1. The **written qualifying exams** on **basic analysis** and **linear algebra** (at undergraduate level) are administered in August (just before fall term begin), January (just before spring term begin) and May (after the end of the spring term).

   - The topics covered by the exams are given in the syllabus lists:
     - Basic analysis
     - Linear algebra
   - **Local Archive** of recent written qualifying exams
   - The results of the August exam can sometimes be used a guide for selecting appropriate courses in the fall semester.
   - Students who do not pass the exam should work to become more prepared to retake the exam either through coursework and faculty mentoring.
   - Students are normally expected to pass both the basic analysis and linear algebra written exams by May of their first year.
   - A Pre-qual Prep (PQP) program is offered to help students review and prepare before the written exams.

   The repository of past exams at the UC-Berkeley Math Dept may be helpful.

2. The **oral qualifying exam** is a two-hour examination on two graduate-level topics given by a committee of two members of the graduate faculty.

   - The two **graduate level topics** for the oral qualifying exam are chosen from the following list of seven:
     - Algebra
     - Complex Analysis
     - Differential Equations
     - Differential Geometry and Topology
     - Probability and Stochastic Processes
     - Real Analysis
     - Scientific Computing

     These links describe syllabus topics and suggested reading for each. The amount of material covered on each topic corresponds roughly to the content of a one semester introductory graduate level course. Many of the typical oral exam questions probe the student's ability to apply the general theory to specific examples.
   - The oral qualifying examination may not be taken before the student has passed the written qualifying examinations.
   - The oral qual is usually taken towards the end of the first year of graduate study.
   - The oral qualifying exam must be taken during a semester for which the student is registered. It may be taken before the last day of final exams of the spring semester. It may also be taken after the summer session starts (which is around 12 days later) so long as the student is registered for the summer session.

- **It must be passed before the beginning of the spring semester of the second year of study.**
- **The process for setting up the oral qual exam:**
  (a) Select two topics from the list of seven.
  (b) Contact the DGS to have the DGS appoint an exam committee. You may list any preferences or requests for Profs. for the exam.
  (c) You will be informed by the DGS on the Profs for the exam. You can then contact them to work out the date and time for the exam. The date of the exam must be during a term (fall, spring, summer) that you are registered for. See the academic calendar.
  (d) While preparing to take the exam, the material on the exam SHOULD be discussed with the members of the committee.
  (e) Practice on the material can be arranged with the committee members or other members of the department.
- Students that do not ultimately pass the oral qualifying exams at Ph.D. level will not continue in the Ph.D. program.

The purpose of the oral qualifying examination is to determine if the student has mastered relevant beginning-graduate coursework at a level needed to continue on to advanced-level coursework and research leading to the Ph.D. (with the preliminary exam coming at that stage). There are several possible outcomes of the oral qualifying exam:

- Passing both topics at the Ph.D. level - the qualifying exam requirements are then complete.
- Falling short of passing one or both topics at the Ph.D. level will be categorized into two levels:
  - Passing at the Master's level.
  - Failing.

  The committee may decide to allow students who do not pass a topic at the Ph.D. level to retake the exam (in front of the full committee) once more.

All students that complete the qualifying exams (at least at Master's level) and have satisfied the course requirement of at least 10 3-unit courses can obtain a Master of Arts degree in Mathematics. Students may earn one Duke MA degree (either in Math or another dept) on route to receiving their Ph.D.

To obtain the MA degree, students must file an "Apply for Graduation" form in the ACES system:

- no later than January 25 for a May degree,
- no later than July 1 for a September degree, and
- no later than November 1 for a December degree

1.2.1 Syllabus for the Qualifying Examination in Basic Analysis

1. Metric Spaces
   (a) Convergence of sequences in metric spaces
   (b) Cauchy sequences
   (c) completeness
   (d) contraction principle
2. Topological spaces
   (a) continuous maps
   (b) Hausdorff spaces
   (c) compactness
   (d) connectedness
3. The real numbers
   (a) The real numbers as a complete ordered field
   (b) closed bounded subsets are compact
   (c) intermediate value theorem
   (d) maxima and minima for continuous functions on a compact set
4. Sequences and series of complex numbers
   (a) standard tests for convergence and divergence of series
   (b) absolute convergence and rearrangements
5. Differentiation
   (a) Differentiation of a function in one real variable
   (b) Mean Value Theorem
   (c) L'Hopital's Rule
   (d) Taylor's Theorem with error estimates
6. Riemann integration of functions in one real variable
   (a) Definition
   (b) Riemann integrable functions
   (c) integration and anti-differentiation
7. Sequences and series of functions
   (a) power series and radii of convergence
   (b) uniform convergence of sequences of functions
   (c) uniform convergence and integration
   (d) integration and differentiation of power series
8. Differential Calculus for functions from n-space to reals and reals to n-space
   (a) Parametrized curves
   (b) tangent vectors
   (c) velocity

(d) acceleration
(e) partial derivatives
(f) directional derivatives
(g) the gradient
(h) the chain rule
(i) Taylor's theorem
(j) local maxima and minima
(k) level surfaces of functions
(l) tangent planes to surfaces in 3-space
(m) Lagrange multipliers

9. Differential Calculus for functions from n-space to m-space
   (a) notion of derivative
   (b) affine function with best approximates a differentiable function at a point
   (c) chain rule
   (d) inverse function theorem
   (e) implicit function theorem

10. Integral Calculus in several variables
    (a) The integral, path and surface integrals
    (b) Green's theorem in the plane
    (c) the divergence theorem in 3-space
    (d) the change of variables formula

References:

- Main reference: first nine chapters of Principles of Mathematical Analysis, 3rd edition, by Walter Rudin.

Other useful references:

- Fleming, Wendell: *Functions of Several Variables*
- Smith, Kennan T.: *Primer of Modern Analysis*
- Edwards, Harold: *Advance Calculus of Several Variables*

1.2.2 Syllabus for the Qualifying Exam in Linear Algebra

1. Fields:
   (a) the field of rational numbers
   (b) the field of real numbers
   (c) the field of complex numbers

2. Linear equations and matrices, reduction to row echelon form.

3. Vector spaces:
   (a) Vector spaces, subspaces, quotient spaces.
   (b) Linearly independent sets.

(c) Linear transformations,
(d) kernel and image,
(e) projections (idempotent linear operators),
(f) the set of linear transformations between two vector spaces forms a vector space.
(g) Bases and dimension for finite dimensional vector spaces.

4. Matrices and linear transformations between finite dimensional vector spaces:
   (a) The matrix of a linear transformation with respect to a choice of bases,
   (b) similarity of matrices and change of basis for linear transformations.
   (c) The inverse of a matrix,
   (d) the determinant of a square matrix,
   (e) the characteristic polynomial,
   (f) the minimal polynomial,
   (g) eigenvectors,
   (h) eigenvalues.
   (i) Diagonalizability,
   (j) Jordan canonical form for square matrices over the complex numbers.
   (k) Cayley-Hamilton theorem.
   (l) Rank + nullity = dimension of domain.
   (m) Dual basis of dual vector space,
   (n) dual of a linear transformation and transpose of a matrix.

5. Finite dimensional inner product spaces:
   (a) Symmetric bilinear forms,
   (b) Hermetian forms,
   (c) non-degeneracy,
   (d) positive definiteness,
   (e) the matrix of a bilinear form.
   (f) Self-adjoint, orthogonal and unitary transformations.
   (g) The standard positive definite inner product on real n-space,
   (h) length and angle,
   (i) diagonalization of real symmetric matrices by unitary matrices.
   (j) Orthogonal projection,
   (k) Gram-Schmidt orthogonalization.

References:

- Lang: *Linear Algebra*
- Strang, Gilbert: *Linear Algebra and its Applications*
- Hoffman, Kenneth and Kunze, Ray: *Linear Algebra*
- Artin, Michael: *Algebra* (Chapters 1,3,4,7)

1.2.3 Syllabus for the Qualifying Examination in Algebra

Groups:

Elementary concepts (homomorphism, subgroup, coset, normal subgroup), solvable groups, commutator subgroup, Sylow theorems, structure of finitely generated Abelian groups. Symmetric, alternating, dihedral, and general linear groups.

Rings:

Commutative rings and ideals (principal, prime, maximal). Integral domains, Euclidean domains, principal ideal domains, polynomial rings, Eisenstein's irreduciblility criterion, Chinese remainder theorem. Structure of finitely generated modules over a principal ideal domain.

Fields:

Extensions: finite, algebraic, separable, inseparable, transcendental, splitting field of a polynomial, primitive element theorem, algebraic closure. Finite Galois extensions and the Galois correspondence between subgroups of the Galois group and subextensions. Solvable extensions and solving equations by radicals. Finite fields.

References:

- Artin, Algebra
- Dummit and Foote, Algebra
- Lang, Algebra
- Hungerford, Algebra

1.2.4 Syllabus for the Qualifying Examination in Complex Analysis

Complex differentiation, Cauchy-Riemann equations, power series, exponential and trigonometric functions.

Cauchy's theorem and integral formula, Cauchy's inequalities, Liouville's theorem, Morera's theorem, classification of isolated singularities, Taylor series, meromorphic functions, Laurent series, fundamental theorem of algebra, residues, winding numbers, argument principle, Rouche's theorem, local behaviour of analytic mappings, open mapping theorem. Harmonic functions, maximum principle, Poisson integral formula, mean value property. Conformal mappings, linear fractional transformations, Schwarz lemma. Infinite products, analytic continuation, multi-valued functions, Schwarz reflection principle, monodromy theorem. Statement and consequences of Riemann mapping theorem and Picard's theorem.

References:

- L. Ahlfors, Complex Analysis
- J. Conway, Functions of One Complex Variable
- Churchill, Complex Variables and Applications
- S. Lang, Complex Analysis
- Levinson and Redheffer, Complex Variables
- Knopp, Theory of Functions, vols I-III.

1.2.5 Syllabus for the Qualifying Exam in Differential Equations

For the qualifying exam in differential equations the candidate is to prepare a syllabus by selecting topics from the list below. The syllabus must be approved by the chairman of the qualifying exam committee and it must be e-mailed to the director of graduate studies for his/her approval. The total amount of material on the syllabus should be roughly equal to that covered in a standard one semester graduate course which has no other graduate course as a prerequisite.

Topics in Ordinary Differential Equations

1. Fundamental existence theorems; uniqueness with the Lipschitz condition; the Gronwall inequality; continuation of solutions to the boundary; dependence on parameters and variational equations.
2. Solution of linear systems of equations with constant coefficients; the exponent of a matrix; the Jordan canonical form and implications for the large time behavior of solutions; classification and phase portraits of 2 by 2 linear systems.
3. The simplest numerical methods, order of accuracy.
4. Equilibria; notion of stable and asymptotically stable equilibria; linearization about an equilibrium; stability of an equilibrium as a consequence of linearized stability; Liapunov functions and their implications for stability.
5. The phase plane; limit cycles; the van der Pol equation; the Poincare-Bendixson Theorem (statement, not proof); the phase portrait of the pendulum equation; strange attractors and the Lorenz system; chaos; bifurcation of equilibria; Hopf bifurcation.

Topics in Partial Differential Equations:

1. Notion of well-posed problem; the classical examples (wave equation, heat equation, Laplace's equation); solution by Fourier series and Fourier transform.
2. First-order equations; geometric interpretation of solutions; method of characteristics for linear and quasilinear equations; domain of dependence and influence; the simplest numerical methods for first-order linear hyperbolic equations; numerical stability and the CFL condition.
3. The wave equation in one space dimension, explicit solution and energy conservation; solution in 3-D by spherical means and 2-D by descent; domain of dependence and influence for the wave equation.
4. The heat equation in free space, fundamental solution, smoothing property; notion of similarity solutions; maximum principle; Duhamel's principle for nonhomogeneous problems.
5. Two-point boundary value problems on an interval, and their Green's functions; Laplace's equation and Poisson's equation; the maximum principle and mean value property for harmonic functions; fundamental solutions of Laplace's equation; representation of solutions by boundary integrals; Dirichlet and Neumann problems; Green's functions (definition, half-space, disk).
6. Notion of distributions, especially the delta function; distributional interpretation of fundamental solutions; weak derivatives; weak formulation of the Dirichlet problem in Hilbert space; eigenvalues of Laplacian in a bounded domain; solution of wave equation or heat equation in a bounded domain by eigenfunction expansions.

1.2.6 Syllabus for the Qualifying Exam in Topology and/or Differential Geometry

For the qualifying exam in topology and/or differential geometry the candidate is to prepare a syllabus by selecting topics from the list below. The syllabus must be approved by the chairman of the qualifying exam committee and it must be e-mailed to the director of graduate studies for his/her approval. The total amount of material on the syllabus should be roughly equal to that covered in a standard one semester graduate course which has no other graduate course as a prerequisite. Example: The syllabus could consist of all topics listed under topology and no other topics.

Topics in Topology

Basic topological notions: Path connectivity, connectivity, product topology, quotient topology. The fundamental group, computation of the fundamental group, van Kampen's theorem, covering spaces. Homology: Singular chains, chain complexes, homotopy invariance, relationship between the first homology and the fundamental group, relative homology, the long exact sequence of relative homology, the Mayer-Vietoris sequence, applications to computing the homology of surfaces, projective spaces, etc. Topological manifolds, differentiable manifolds.

References:

- Harper and Greenberg, Algebraic Topology, a First Course, parts I and II

Topics in the Differential Geometry of Curves and Surfaces in Euclidean Space

The orthogonal group in 2 and 3 dimensions, the Serret-Frenet frame of a space curve; the Gauss map and the Weingarten equation for a surface in Euclidean 3-space, the Gauss curvature equation and the Codazzi-Mainardi equation for a surface in Euclidean 3-space; the surfaces in Euclidean 3-space of zero Gauss curvature; the fundamental existence and rigidity theorem for surfaces in Euclidean space; the Gauss-Bonnet formula for surfaces in Euclidean 3-space.

References:

- M. do Carmo, Differential Geometry of Curves and Surfaces

Topics in the Differential Geometry of Riemannian Manifolds

Riemannian metrics and connections; geodesics and the first and second variational formulas; completeness and the Hopf-Rinow theorem; the Riemann curvature tensor, sectional curvature, Ricci curvature, and scalar curvature; the theorems of Hadamard and Bonnet-Myers; the Jacobi equation; the geometry of submanifolds — the second fundamental form, equations of Gauss, Ricci, and Codazzi; spaces of constant curvature.

References:

- M. do Carmo, Riemannian Geometry
- M. Spivak, A Comprehensive Introduction to Differential Geometry
- S. Helgason, Differential Geometry, Lie Groups, and Symmetric Spaces
- S. Sternberg, Lectures on Differential Geometry, 2nd ed.

1.2.7 Syllabus for the Qualifying Exam in Probability and Stochastic Processes

Undergraduate Material

It will be expected that the candidate knows material from a standard undergraduate, post-calculus level course in probability. Hence it is expected that the candidate knows the following:

- Basic notions of probability and conditional probability including Bayes rule
- Discrete Probability Densities (binomial, Poisson, geometric, hypergeometric)
- Continuous Probability Densities (normal, exponential, uniform, joint normal)
- Expectation, Variance, Standard Deviation, Covariance
- Poisson approximation to binomial and normal approximations (central limit theorem)
- Chebyshev's inequality and Weak Law of Large Numbers

Graduate Material

For the exam, the student can choose one of two tracks. Track I consists primarily of measure theoretic probability and corresponds to Mathematics 287. Track II consists primarily of stochastic processes from a non-measure theoretic perspective and corresponds to Mathematics 216.

Core Material (Required for either Track)

Measure theoretic foundations of probability theory: what is a probability space?; -algebras; random variables as measurable functions; notions of convergence (almost sure versus in probability)

Finite Markov chains in discrete time (recurrence classes, periodicity, convergence to invariant probability)

Track I (Measure Theoretic Probability)

Borel-Cantelli Lemma; Zero-One Law

Expectation and other moments; convergence in ; weak law of large numbers ; strong law of large numbers (with idea of proof) ; law of iterated logarithm (without proof)

weak convergence of probability measures; characteristic functions of random variables and their relationship to weak convergence; central limit theorem with proof (in i.i.d. case)

conditional expectation; martingales (in discrete time); optional sampling theorem; martingale convergence theorem

Definition of Brownian motion; how is it constructed?; non-differentiability; strong Markov property

Track II

Markov chains with infinite state space: positive recurrence, null recurrence transience; simple random walk; reversible Markov chains and relationship between eigenvalues and convergence to equilibrium

Markov chains with continuous time: generator, relationship to discrete time models; examples: Poisson processes, Markovian queues

Branching processes

conditional expectation; martingales (in discrete time); optional sampling theorem; martingale convergence theorem

Brownian motion and stochastic integration from a non-measure theoretic perspective; Ito's formula

1.2.8 Syllabus for the Qualifying Examination in Real Analysis

Outer measure, measurable sets, sigma-algebras, Borel sets, measurable functions, the Cantor set and function, non-measurable sets.

Lebesgue integration, Fatou's Lemma, the Monotone Convergence Theorem, the Lebesgue Dominated Convergence Theorem, convergence in measure.

L^p spaces, Holder and Minkowski inequalities, completeness, dual spaces.

Abstract measure spaces and integration, signed measures, the Hahn decomposition, the Radon-Nikodym Theorem, the Lebesgue Decomposition Theorem.

Product measures, the Fubini and Tonelli Theorems, Lebesgue measure on real n-space.

Equicontinuous families, the Ascoli-Arzela Theorem.

Hilbert spaces, orthogonal complements, representation of linear functionals, orthonormal bases.

References:

- H. L. Royden, Real Analysis, Chap. 1 - 7, 11, 12.
- M. Reed and B. Simon, Methods of Mathematical Physics I: Functional Analysis, chapters 1,2.
- G. B. Folland, Real Analysis, Chap. 0 - 3, 6.

1.2.9 Syllabus for the Qualifying Examination in Scientific Computing

For the qualifying exam in scientific computing the candidate is to prepare a syllabus by selecting topics from the list below. The syllabus must be approved by the chairman of the qualifying exam committee and it must be e-mailed to the director of graduate studies for his/her approval. The total amount of material on the syllabus should be roughly equal to that covered in a standard one semester graduate course which has no other graduate course as a prerequisite.

1. Hardware/Programming issues: Machine numbers, floating point arithmetic, accumulation of rounding errors, memory hierarchy, arrays in C and FORTRAN, C++ scope, C++ classes, organization of loops for computational efficiency.
2. Computational linear algebra: Basic linear algebra, solution of linear equations: direct and iterative methods, convergence, matrix factorizations (LU, LL^T, QR, SVD), linear equations and least squares, eigenvalues and eigenvectors.
3. Iterative methods for nonlinear equations: Fixed point theorems, Convergence proofs, linear iteration methods, Newton and secant methods for scalar equations, techniques for enhancing global convergence, Newton and quasi-Newton methods for nonlinear systems.
4. Approximation theory and interpolation: Interpolating polynomials, Lagrange and Newton interpolation, divided differences, piecewise polynomial approximation, least squares polynomial approximation, orthogonal decompositions: Fourier series/transforms and orthogonal polynomials.
5. Differentiation and integration: Divided differences, Richardson extrapolation, midpoint and trapezoidal rules, the Euler-Maclaurin formula, Gaussian quadrature, singular integrals.
6. Initial value problems for ordinary differential equations: Finite difference methods: order of accuracy, stability analysis, convergence results, Euler's explicit and implicit methods, local truncation errors/rounding errors/accumulated errors, higher order methods: Adams Bashforth and Adams Moulton methods, Runge-Kutta methods, backward differentiation formulas, stiffness.
7. Boundary value problems for ordinary differential equations: Shooting methods, finite difference methods, finite element methods, eigenvalue problems.

References:

- An Introduction to Numerical Analysis (2nd ed), by K. E. Atkinson, Wiley, New York, 1989.
- Analysis of Numerical Methods, by E. Isaacson and H. B. Keller, Dover, New York, 1994.
- Numerical Analysis: Mathematics of Scientific Computing, by David R. Kincaid, E. Ward Cheney, and Ward Cheney
- Introduction to Numerical Analysis, by J. Stoer and R. Bulirsch
- Scientific Computing, by John Trangenstein (available online)

1.3 Mentors and Advisors

Graduate school is fundamentally different from undergraduate study. Like college, graduate school begins with taking courses. Graduate courses are generally more challenging, demanding and rigorous. They are focus on more advanced and specialized topics and seminars will involve more direct student-instructor interaction, for example in discussing challenging homework problems or new research directions in the area of study. But beyond taking courses, graduate study is about using your skills in an area of mathematics to produce a new contribution to science and mathematics. This creative pursuit of novel results directly leads to your doctoral thesis and your Ph.D.

At every point in your career as a graduate student at Duke, you will receive guidance for the steps in this process. On entering the program, the Director of Graduate Studies will assign to you a faculty member to serve as your **academic mentor**.

- **Academic mentors** serve as general advisors, recommending which courses which might best fit your research goals. Mentors will help you prepare for the qualifying exams. The mentor continues to offer guidance to students until they have selected a research advisor.

- **Research advisors** help students select a specialized area of mathematical study to focus on and a problem that ultimately serves as the basis of the student's thesis. The research advisor helps arrange the preliminary exam and the final thesis defense.

1.4 The Preliminary Exam

The primary purpose of the preliminary examination is to determine if the student has acquired the specialized knowledge necessary for their proposed dissertation research.

The preliminary exam is part of the Graduate School Requirements for the Ph.D. Degree. For more details on University requirements, see the Graduate School's rules governing the exam.

The key regulations on the exam are:

1. The preliminary exam is an oral exam in front of a committee of at least four professors.

2. The chair of the committee should be the student's research advisor.

3. The exam must be taken **before the end of the student's third year** (see Grad School Rules).

   A student who has not passed the examination by this time must file with the Dean of the Graduate School a statement, approved by the Director of Graduate Studies, explaining the delay and setting a date for the examination. Except under unusual circumstances, extension will not be granted beyond the middle of the fourth year.

   A student **must be registered during the term** in which they takes the preliminary examination. In the summer a preliminary examination may be scheduled only between the opening and closing dates of the summer session. See the academic calendar.

4. The committee is recommended by the student and their advisor to the DGS who appoints the committee. The DGS must submit the list of committee members on a committee approval form to the Graduate School. After the Graduate School approves the committee, there is a **60 day waiting period** before the exam can take place!
   Students should NOT delay the start of the committee approval process: only a list of Professors is needed (not an exam date or time or syllabus!)

5. Professors from other Universities can serve on prelim committees with the approval of the Graduate School.

6. The student and advisor will write a **syllabus** to be covered in the exam. This outline must be approved by the members of the exam committee and the DGS. The student and the committee should agree upon a syllabus for both the major and minor topics. These must be sufficiently separated. The syllabus must be **submitted to the Director of Graduate Studies for approval**.

7. Unless changes take place later, the committee for the preliminary exam will also be the committee for the PhD thesis defense.

Satisfactory completion of the preliminary examination

This occurs when at least 3 of the 4 committee members, one of whom is the chair, cast an affirmative vote. (For a 5 member committee, at least 4 affirmative votes are required.) See the Graduate School Requirements

1.5 Thesis, Final defense, and Graduation

(updated Oct 16, 2010) The Duke Graduate School has **rigid rules** regarding the thesis, final defense and graduation.
These are best understood described **in reverse chronological order**, starting from the successful conclusion of graduate school:

- The Ph.D. degree is officially conferred at one of only **three** official graduation dates:
  - May (after the spring semester)
  - September (after the summer semester)
  - December (after the fall semester)
- The final version of the dissertation (approved by the defense committee, with all corrected completed within thirty days of the final defense) must be turned in by the **Graduate Degree deadline dates** for the desired degree conferral dates. The final defense should be scheduled at least one week before the deadline date.
- **AT LEAST SEVEN DAYS** before the defense, the following must be completed:
  - Copies of the dissertation are given to all committee members for them to read.
  - A public copy of the thesis is put in the department lounge.
  - The thesis advisor sends a letter/email to the Graduate School indicating their approval of the readiness of the thesis for defense.
  - DGS signs approval on the public announcement of the final defense and the Graduate School approves the defense.
- **AT LEAST TWO WEEKS** before the defense (earlier if close to the degree deadlines), a version of the thesis must be submitted for the "initial electronic dissertation submission" (format check).
  This must be done before scheduling an appointment to pick up the final examination paperwork!
- Students must file an Apply for Graduation Form for the graduate school in the ACES system.
  - no later than January 25 for a May degree,
  - no later than July 1 for a September degree,
  - no later than November 1 for a December degree,
  - no later than one month prior to defense, and
  - no earlier than the beginning of the semester in which they will defend their thesis.

  If the defense is delayed, this form must be re-submitted!
- The default committee for the thesis defense is the committee from the preliminary exam.
  Any requests for changes to the committee **MUST BE APPROVED** by the Graduate School based on a committee approval form to be filed by the DGS **AT LEAST 60 DAYS BEFORE THE DEFENSE**
- Students must be registered for the semester during which their defense will take place: fall, spring, or summer-full.
- Students must have completed all of the Graduate School's Ph.D. Requirements before the Ph.D. degree can be awarded. Most major among these are:
  - The process connected with the final thesis defense.
  - Passing the Prelim exam
  - RCR: 12 credit hours on Responsible Conduct of Research
  - EIS: English for International Students (as appropriate)

YOU are responsible for planning ahead (with appropriate timing) for all of these steps.

Important links

- Graduation Deadline Dates
- Graduation process
- Thesis check appointment
- Thesis Defense and Graduation
- Guide for preparing your thesis
- Apply for Graduation Form
- Ph.D. Requirements

For further information or other questions, contact the DGS!

1.6 Duties of Graduate Students with Teaching Assistantships

- All entering graduate students are required to attend a one-week teacher training program which is usually scheduled to begin at 9:00 a.m. on Monday of the week before fall classes begin. This program is designed to prepare graduate students to lead calculus labs and to begin the training for teaching a laboratory calculus course or other introductory calculus course. To see a detailed schedule for the most recent (or next) training week, refer to the training week schedule.
- All first-year graduate students will participate in a year-long teacher training program which is run by Jack Bookman of the Math Department, and which will include seminars, observing experienced teachers, practice teaching while being video-taped, practice grading, practice test writing, and guidance on holding conferences with undergraduates. After a graduate student begins teaching (normally in the second year), a faculty member will periodically visit the class and provide feedback to the teaching assistant. For more details about this training you can read Jack Bookman's teacher training outline.
- First-year graduate students usually are assigned the job of leading or assisting with a calculus lab. Each lab meets once a week for one hour and forty-five minutes. In these labs we use a locally written lab manual and we ask students to use the TI-83 calculator (or one with similar capabilities). Lab assistants will also grade papers from labs, participate in the help-room, and help with the grading of the Departmental final exams at the end of the semester.
- Once a graduate student has been assigned to teach a class, then that teacher will have the same duties that all teachers in our courses assume. In addition to the usual lesson writing, lecturing, and grading, these duties also entail participation (for 2 hours a week) in a Departmental help room. All teachers help to grade the departmental exams at the end of the semester.

Training for Teaching Assistants at Duke

Learning to teach is an important part of the education of our mathematics graduate students and being a teaching assistant is an important part of both their professional development and financial support. Mathematics graduate students typically begin their teaching responsibilities during their first year of graduate study when they serve as lab assistants and work in the help room. Beginning in their second year, they teach their own section of 20-35 students, typically a calculus class meeting 3 hours a week, with a weekly laboratory session supervised by two teaching assistants. The teacher training program for graduate students has been ongoing since fall, 1987. The program is coordinated by Jack Bookman, a full-time instructor in the mathematics department, in consultation with the Director of Graduate Studies and the Supervisor of First-year Instruction.

During the week before classes begin in the fall, the graduate students who will be serving as lab assistants participate in a week-long workshop led by Lewis Blake, Supervisor of First-year Instruction. In this workshop the participants are introduced to Duke's laboratory calculus course. This workshop is designed to enable graduate students to begin their work as lab assistants.

During their first year of graduate study, all graduate students participate in a weekly teaching seminar led by Jack Bookman. There are two related purposes of the seminar: (1) to prepare the graduate students to teach introductory calculus courses here at Duke and (2) to introduce the graduate students to some of the educational issues that they will need to know about and act on if they are to become effective college mathematics faculty. The activities of the seminar include:

1. A discussion of what constitutes good teaching and how undergraduates learn mathematics.
2. Observations of lessons taught by experienced teachers.
3. Discussion of observations.
4. How to organize lessons: planning, time management, homework .
5. Overview of content of our Calculus courses with emphasis on what students find difficult.

6. Making up hour exams.
7. Grading exams.
8. Current issues in undergraduate mathematics education.
9. Meeting of first-year graduate students with the graduate students teaching for the first time to discuss the problems of first year teachers.
10. Office hours, how to start the semester, rules and regulations, services available to freshmen.
11. Presentation of a 15-minute practice lesson.
12. Two lectures given to real calculus classes. These presentations are observed by the department's coordinator of teacher training or a faculty member designated by him. There is a follow-up discussion. and when possible, the students who were taught by the graduate student complete a short evaluation consisting of three questions: What was best about the instruction?; What was worst?; What would be one suggestion you would make to the TA to improve the TA's teaching?

Most of our graduate students begin teaching their own classes (usually a section of Calculus I or II) during the fall of their second year. During the semester that a graduate student begins teaching his or her own classes, they are observed twice–once during the second week and once during the third week. These observations are followed up with a discussion. The observations are made by various regular rank, tenured faculty and by full time instructors. If the quality of teaching is satisfactory, no more observations are made, but if problems are perceived, another observation will be made to see if the suggestions are being implemented. At the end of that first semester of teaching and after the graduate student reads his or her students' Teacher-Course Evaluations, the graduate student will write a self-evaluation describing his or her perceived strengths and weaknesses and discussing ways to improve. The Teacher-Course Evaluations and the self-evaluations serve as the basis for a discussion between the new teacher and the coordinator of teacher training. If there are no major problems, this point marks the end of their training. If problems are evident, a program is designed to help that individual graduate student improve his or her teaching.

Teaching links

- Grad Handbook, section on Professional Development
- L. P. Smith Awards for teaching excellence

1.7 Graduate Student Annual Reports

The following is an excerpt from the Graduate School Handbook for Directors of Graduate Study:

> *... all doctoral students ... at Duke University will be required, as a condition of their enrollment, to file annually with their Director of Graduate Studies a written report on their progress towards the Ph.D. degree. This report will be due on **April 15** of each year a student is enrolled in the Graduate School. The Director of Graduate Studies will subsequently certify to the Associate Dean of the Graduate School that this report has been received and will, where deemed appropriate, forward copies to the student's doctoral committee. The Graduate School will track the submission of the annual reports as a "milestone" in the student's official record. Failure to submit this annual progress report will jeopardize a student's continuation in the graduate program.*
>
> *For students who have not completed coursework or qualifying/preliminary examinations, this report should identify the likely schedule of courses still to be taken, and the likely dates at which the student will sit for the Qualifying and/or Preliminary examinations. For students who have passed the Preliminary examinations, the report should specify annually the progress of their dissertation research, identify any portions of completed written work, establish a clear time-line for completion of any remaining chapters of the dissertation, and set a target date for final defense.*

The Mathematics Department has the following requirements for graduate students in Mathematics:

- Write a 1 to 2 page summary of your academic activities and progress in the Graduate Program.
- Submit the report to your mentor or advisor for feedback and revisions to be made.
- When your faculty supervisor has approved and signed the final version of your report, forward it to the DGS by the **April 15 deadline**.

The contents of the report should include:

- **First Year** students will file a report listing:
  - Courses taken in year one (both fall and spring)
  - Likely courses to be taken in year two
  - Status of their qualifying exam:
    - The written qualifying examination dates and outcomes
    - The oral qualifying exam topics, syllabus and committee members
  - Plans for summer activities: research, summer programs, ...
- **Second Year** students will file a report listing:
  - Courses taken in year two (both fall and spring)
  - Any remaining courses to be taken in year three
  - Oral qualifying exam committee members, date(s) and results
  - Research directions and planned research advisor
  - Progress towards the prelim exam
  - Plans for summer activities: research, summer programs, conferences, ...
- **Third Year** students will file a report listing:
  - Any remaining courses to be taken
  - Status of the preliminary exam

    * chair and committee members for the preliminary exam
    * preliminary exam date and outcome
  - research plans and progress
    * Thesis advisor
    * Proposed thesis topic and brief outline of initial work
  - Plans for summer research, summer programs, conferences, ...

- **Fourth and Higher Years** students will file a report listing:
  - Courses taken, if any
  - Thesis advisor and thesis defense committee members
  - Planned thesis title
  - Outline of progress on thesis research
  - Summary of any portions of completed written work
  - Timeline for completion of remaining thesis chapters
  - Target date for final defense

Students should will include information about any activities related to research in their annual report:

- Talks or posters presented (for example, in the graduate student seminar, or at conferences, meetings)
- Conferences attended
- Collaborations with post doctoral fellows and/or faculty on research projects or departmental activities
- Publications
- Summer programs, summer schools, fellowships, grants, ...
- Departmental and University activities (seminars, tea, grading, teaching, tutoring, student representatives, student government, ...)

1.8 Duke Graduate School Policies

The rules set by the Duke Graduate School are the "supreme law of the land" and may only be supplemented but never over-ridden by any departmental policies. When in doubt on the rules for any situation not covered in the Math Grad Webpages, consult the DGS, or see the Graduate School's webpages:

Important top-level links

- Duke University Graduate School
- Duke Graduate School rules, requirements, and information
  - Graduate Student Handbook
  - Graduate School Course Bulletin
  - Academic calendar
- Grad Handbook, section on Academics
- Rules on courses
  - Duke Registrar Information
  - Academic Calendar
  - ACES FAQ
  - ACES Login
- EIS: English for International Students
  - Graduate School EIS Requirements
  - EIS Placement exams
  - EIS Courses
- RCR: Responsible Conduct of Research
  - RCR: Natural Science Track Requirements
  - Natural Sciences Orientation
  - RCR Forums

Chapter 2

Courses

2.1 Typical Graduate Mathematics Course sequences

The following are typical graduate course sequences for different focus areas in mathematics. For Fall of first year 3 courses are recommended and Math 390, and 4 courses in the spring semester.

Applied Mathematics/Numerical Analysis

- **First year**
  Fall
  - Math 224 Scientific Computing I
  - Math 231 ODE
  - Math 241 Real Analysis

  Spring
  - Math 225 Scientific Computing II
  - Math 232 Intro to PDE
  - Math 228 Mathematical Fluid Dynamics
  - Math 233 Asymptotics and Perturbation Methods
  - Math 229 Mathematical Modeling
  - Math 287 Probability Theory
- **Second year**
  Fall
  - Math 216 Applied Stochastic Processes
  - Math 226 Numerical PDE I
  - Math 233 Asymptotics and Perturbation Methods
  - Math 282 Elliptic PDE
  - Math 288 Topics in Probability

  Spring
  - Math 227 Numerical PDE II
  - Math 228 Mathematical Fluid Dynamics
  - Math 229 Mathematical Modeling
  - Math 281 Hyperbolic PDE
  - Math 287 Probability Theory

Analysis/Probability

- **First year**
  Fall
  - Math 215 Mathematical Finance
  - Math 216 Applied Stochastic Processes
  - Math 231 ODE
  - Math 241 Real Analysis

  Spring
  - Math 219 Stochastic Calculus
  - Math 232 Intro to PDE
  - Math 245 Complex Analysis
  - Math 287 Probability Theory
- **Second year**
  Fall
  - Math 215 Mathematical Finance
  - Math 216 Applied Stochastic Processes
  - Math 242 Functional Analysis
  - Math 282 Elliptic PDE
  - Math 288 Topics in Probability theory

  Spring
  - Math 219 Stochastic Calculus
  - Math 249 Topics in Functional Analysis
  - Math 287 Probability Theory
  - Math 233 Asymptotics and Perturbation Methods

Algebra/Geometry/Topology

- **First year**
  Fall
  - Math 231 ODE
  - Math 241 Real Analysis
  - Math 251 Groups, Rings and Fields

  Spring
  - Math 245 Complex Analysis
  - Math 252 Commutative Algebra
  - Math 261 Algebraic Topology I
  - Math 267 Differential Geometry
- **Second year**
  Fall
  - Math 253 Representation Theory
  - Math 262 Algebraic Topology II
  - Math 272 Riemann Surfaces
  - Math 282 Elliptic PDE

  Spring
  - Math 268 Topics in Differential Geometry
  - Math 273 Algebraic Geometry
  - Math 274 Number Theory
  - Math 236 General Relativity

Geometry/Mathematical Physics

- **First year**
  Fall
  - Math 231 ODE
  - Math 241 Real Analysis
  - Math 251 Groups, Rings and Fields

  Spring
  - Math 245 Complex Analysis
  - Math 252 Commutative Algebra
  - Math 261 Algebraic Topology I
  - Math 267 Differential Geometry
- **Second year**
  Fall
  - Math 236 General Relativity
  - Math 253 Representation Theory
  - Math 272 Riemann Surfaces
  - Math 282 Elliptic PDE
  - Ph 341 Quantum Field Theory

  Spring
  - Math 268 Topics in Differential Geometry
  - Math 273 Algebraic Geometry
  - Math 274 Number Theory
  - Math 262 Algebraic Topology II
  - Math 274 Number Theory

2.2 Course list from the Graduate School Bulletin

Mathematics (MATH)
200. Introduction to Algebraic Structures I. Groups: symmetry, normal subgroups, quotient groups, group actions. Rings: homomorphisms, ideals, principal ideal domains, the Euclidean algorithm, unique factorization. Not open to students who have had Mathematics 121. Prerequisite: Mathematics 104 or equivalent. Instructor: Staff. 3 units.
201. Introduction to Algebraic Structures II. Fields and field extensions, modules over rings, further topics in groups, rings, fields, and their applications. Prerequisite: Mathematics 200, or 121 and consent of instructor. Instructor: Staff. 3 units.
203. Basic Analysis I. Topology of Rn, continuous functions, uniform convergence, compactness, infinite series, theory of differentiation, and integration. Not open to students who have had Mathematics 139. Prerequisite: Mathematics 104. Instructor: Staff. 3 units.
204. Basic Analysis II. Differential and integral calculus in Rn. Inverse and implicit function theorems. Further topics in multivariable analysis. Prerequisite: Mathematics 104; Mathematics 203, or 139 and consent of instructor. Instructor: Staff. 3 units.
205. Topology. Elementary topology, surfaces, covering spaces, Euler characteristic, fundamental group, homology theory, exact sequences. Prerequisite: Mathematics 104. Instructor: Staff. 3 units.
206. Differential Geometry. Geometry of curves and surfaces, the Serret-Frenet frame of a space curve, Gauss curvature, Cadazzi-Mainardi equations, the Gauss-Bonnet formula. Prerequisite: Mathematics 104. Instructor: Staff. 3 units.
211. Applied Partial Differential Equations and Complex Variables. Initial and boundary value problems for the heat and wave equations in one and several dimensions. Fourier series and integrals, eigenvalue problems. Laplace transforms, solutions via contour integration, and elementary complex variables. Solutions via Green's functions. Intended for applied math students and students in science and engineering. Prerequisite: Mathematics 107 and 108 or the equivalent. Instructor: Staff. 3 units.
214S. Modeling of Biological Systems. Research seminar on mathematical methods for modeling biological systems. Exact content based on research interests of students. Review methods of differential equations and probability. Discuss use of mathematical techniques in development of models in biology. Student presentations and class discussions on individual research projects. Presentation of a substantial individual modeling project to be agreed upon during the first weeks of the course. May serve as capstone course for MBS certificate. Not open to students who have had MBS 200S. Prerequisites: Mathematics 107 or 131 or consent of instructor. 1 unit. C-L: Modeling Biological Systems 214S, Computational Biology and Bioinformatics 230S
215. Mathematical Finance. An introduction to the basic concepts of mathematical finance. Topics include modeling security price behavior, Brownian and geometric Brownian motion, mean variance analysis and the efficient frontier, expected utility maximization, Ito's formula and stochastic differential equations, the Black-Scholes equation and option pricing formula. Prerequisites: Mathematics 103, 104, 135 or equivalent, or consent of instructor. Instructor: Staff. 3 units. C-L: Economics 225
216. Applied Stochastic Processes. An introduction to stochastic processes

without measure theory. Topics selected from: Markov chains in discrete and continuous time, queuing theory, branching processes, martingales, Brownian motion, stochastic calculus. Prerequisite: Mathematics 135 or equivalent. Instructor: Staff. 3 units. C-L: Statistics and Decision Sciences 253

217. Linear Models. 3 units. C-L: see Statistics and Decision Sciences 244

219. Introduction to Stochastic Calculus. Introduction to the theory of stochastic differential equations oriented towards topics useful in applications. Brownian motion, stochastic integrals, and diffusions as solutions of stochastic differential equations. Functionals of diffusions and their connection with partial differential equations. Ito's formula, Girsanov's theorem, Feynman-Kac formula, Martingale representation theoerm. Additional topics have included one dimensional boundary behavior, stochastic averaging, stochastic numerical methods. Prerequisites: Undergraduate background in real analysis (Mathematics 139) and probability (Mathematics 135). Instructor: Staff. 3 units.

221. Numerical Analysis. 3 units. C-L: see Computer Science 250; also C-L: Statistics and Decision Sciences 250

224. Scientific Computing. Structured scientific programming in C/C++ and FORTRAN. Floating point arithmetic and interactive graphics for data visualization. Numerical linear algebra, direct and iterative methods for solving linear systems, matrix factorizations, least squares problems and eigenvalue problems. Iterative methods for nonlinear equations and nonlinear systems, Newton's method. Prerequisite: Mathematics 103 and 104. Instructor: Staff. 3 units.

225. Scientific Computing II. Approximation theory: Fourier series, orthogonal polynomials, interpolating polynomials and splines. Numerical differentiation and integration. Numerical methods for ordinary differential equations: finite difference methods for initial and boundary value problems, and stability analysis. Introduction to finite element methods. Prerequisite: Mathematics 224 and familiarity with ODEs at the level of Mathematics 107 or 131. Instructor: Staff. 3 units.

226. Numerical Solution of Hyperbolic Partial Differential Equations. Numerical solution of hyperbolic conservation laws. Conservative difference schemes, modified equation analysis and Fourier analysis, Lax-Wendroff process. Gas dynamics and Riemann problems. Upwind schemes for hyperbolic systems. Nonlinear stability, monotonicity and entropy; TVD, MUSCL, and ENO schemes for scalar laws. Approximate Riemann solvers and schemes for hyperbolic systems. Multidimensional schemes. Adaptive mesh refinement. Prerequisite: Mathematics 224, 225, or consent of instructor. Instructor: Staff. 3 units.

227. Numerical Solution of Elliptic and Parabolic Partial Differential Equations. Numerical solution of parabolic and elliptic equations. Diffusion equations and stiffness, finite difference methods and operator splitting (ADI). Convection-diffusion equations. Finite element methods for elliptic equations. Conforming elements, nodal basis functions, finite element matrix assembly and numerical quadrature. Iterative linear algebra; conjugate gradients, Gauss-Seidel, incomplete factorizations and multigrid. Mixed and hybrid methods. Mortar elements. Reaction-diffusion problems, localized phenomena, and adaptive mesh refinement. Prerequisite: Mathematics 224, 225, or consent of instructor. Instructor: Staff. 3 units.

228. Mathematical Fluid Dynamics. Properties and solutions of the Euler and Navier-Stokes equations, including particle trajectories, vorticity, conserved quantities, shear, deformation and rotation in two and three

dimensions, the Biot-Savart law, and singular integrals. Additional topics determined by the instructor. Prerequisite: Mathematics 133 or 211 or an equivalent course. Instructor: Staff. 3 units.
229. Mathematical Modeling. Formulation and analysis of mathematical models in science and engineering. Emphasis on case studies; may include individual or team research projects. Instructor: Staff. 3 units.
231. Ordinary Differential Equations. Existence and uniqueness theorems for nonlinear systems, well-posedness, two-point boundary value problems, phase plane diagrams, stability, dynamical systems, and strange attractors. Prerequisite: Mathematics 104, 107 or 131, and 203 or 139. Instructor: Staff. 3 units.
232. Introduction to Partial Differential Equations. Fundamental solutions of linear partial differential equations, hyperbolic equations, characteristics, Cauchy-Kowalevski theorem, propagation of singularities. Not open to students who have taken the former Mathematics 297. Prerequisite: Mathematics 204 or equivalent. Instructor: Staff. 3 units.
233. Asymptotic and Perturbation Methods. Asymptotic solution of linear and nonlinear ordinary and partial differential equations. Asymptotic evaluation of integrals. Singular perturbation. Boundary layer theory. Multiple scale analysis. Prerequisite: Mathematics 108 or equivalent. Instructor: Staff. 3 units.
236. General Relativity. 3 units. C-L: see Physics 292
241. Real Analysis. Measures; Lebesgue integral; Lk spaces; Daniell integral, differentiation theory, product measures. Prerequisite: Mathematics 204 or equivalent. Instructor: Staff. 3 units.
242. Functional Analysis. Metric spaces, fixed point theorems, Baire category theorem, Banach spaces, fundamental theorems of functional analysis, Fourier transform. Prerequisite: Mathematics 241 or equivalent. Instructor: Staff. 3 units.
245. Complex Analysis. Complex calculus, conformal mapping, Riemann mapping theorem, Riemann surfaces. Prerequisite: Mathematics 204 or equivalent. Instructor: Staff. 3 units.
250. Computation in Algebra and Geometry. Application of computing to problems in areas of algebra and geometry, such as linear algebra, algebraic geometry, differential geometry, representation theory, and number theory, use of general purpose symbolic computation packages such as Maple or Mathematica; use of special purpose packages such as Macaulay, PARI-GP, and LiE; programming in C/C++. Previous experience with programming or the various mathematical topics not required. Corequisite: Mathematics 251 or consent of instructor. Instructor: Staff. 3 units.
251. Groups, Rings, and Fields. Groups including nilpotent and solvable groups, p-groups and Sylow theorems; rings and modules including classification of modules over a PID and applications to linear algebra; fields including extensions and Galois theory. Prerequisite: Mathematics 201 or equivalent. Instructor: Staff. 3 units.
252. An Introduction to Commutative Algebra and Algebraic Geometry. Affine algebraic varieties, Groebner bases, localization, chain conditions, dimension theory, singularities, completions. Prerequisite: Mathematics 251 or equivalent. Instructor: Staff. 3 units.
253. Representation Theory. Representation theory of finite groups, Lie algebras and Lie groups, roots, weights, Dynkin diagrams, classification of semisimple Lie algebras and their representations, exceptional groups, examples and applications to geometry and mathematical physics.

Prerequisite: Mathematics 200 or equivalent. Instructor: Staff. 3 units.
C-L: Physics 293
261. Algebraic Topology I. Fundamental group and covering spaces, singular and cellular homology, Eilenberg-Steenrod axioms of homology, Euler characteristic, classification of surfaces, singular and cellular cohomology. Prerequisite: Mathematics 200 and 205 or consent of instructor. Instructor: Staff. 3 units.
262. Algebraic Topology II. Universal coefficient theorems, Knneth theorem, cup and cap products, Poincar duality, plus topics selected from: higher homotopy groups, obstruction theory, Hurewicz and Whitehead theorems, and characteristic classes. Prerequisite: Mathematics 261 or consent of instructor. Instructor: Staff. 3 units.
263. Topics in Topology. Algebraic, geometric, or differential topology. Consent of instructor required. Instructor: Staff. 3 units.
264. Computational Topology. 3 units. C-L: see Computer Science 236
267. Differential Geometry. Differentiable manifolds, fiber bundles, connections, curvature, characteristic classes, Riemannian geometry including submanifolds and variations of length integral, complex manifolds, homogeneous spaces. Prerequisite: Mathematics 204 or equivalent. Instructor: Staff. 3 units.
268. Topics in Differential Geometry. Lie groups and related topics, Hodge theory, index theory, minimal surfaces, Yang-Mills fields, exterior differential systems, harmonic maps, symplectic geometry. Prerequisite: Mathematics 267 or consent of instructor. Instructor: Staff. 3 units.
272. Riemann Surfaces. Compact Riemann Surfaces, maps to projective space, Riemann-Roch Theorem, Serre duality, Hurwitz formula, Hodge theory in dimension one, Jacobians, the Abel-Jacobi map, sheaves, Cech cohomology. Prerequisite: Mathematics 245 and Mathematics 261 or consent of instructor. Instructor: Staff. 3 units.
273. Algebraic Geometry. Projective varieties, morphisms, rational maps, sheaves, divisors, sheaf cohomology, resolution of singularities. Prerequisite: Mathematics 252 and 272; or consent of instructor advised. Instructor: Staff. 3 units.
274. Number Theory. Binary quadratic forms; orders, integral closure; Dedekind domains; fractional ideals; spectra of rings; Minkowski theory; fundamental finiteness theorems; valuations; ramification; zeta functions; density of primes in arithmetic progressions. Prerequisites: Mathematics 201 or 251 or consent of instructor. Instructor: Staff. 3 units.
277. Topics in Algebraic Geometry. Schemes, intersection theory, deformation theory, moduli, classification of varieties, variation of Hodge structure, Calabi-Yau manifolds, or arithmetic algebraic geometry. Prerequisite: Mathematics 273 or consent of instructor. Instructor: Staff. 3 units.
278. Topics in Complex Analysis. Geometric function theory, function algebras, several complex variables, uniformization, or analytic number theory. Prerequisite: Mathematics 245 or equivalent. Instructor: Staff. 3 units.
281. Hyperbolic Partial Differential Equations. Linear wave motion, dispersion, stationary phase, foundations of continuum mechanics, characteristics, linear hyperbolic systems, and nonlinear conservation laws. Prerequisite: Mathematics 232 or equivalent. Instructor: Staff. 3 units.
282. Elliptic Partial Differential Equations. Fourier transforms, distributions, elliptic equations, singular integrals, layer potentials, Sobolev spaces, regularity of elliptic boundary value problems.

Prerequisite: Mathematics 232 and 241 or equivalent. Instructor: Staff. 3 units.

283. Topics in Partial Differential Equations. Hyperbolic conservation laws, pseudo-differential operators, variational inequalities, theoretical continuum mechanics. Prerequisite: Mathematics 281 or equivalent. Instructor: Staff. 3 units.

287. Probability. Theoretic probability. Triangular arrays, weak laws of large numbers, variants of the central limit theorem, rates of convergence of limit theorems, local limit theorems, stable laws, infinitely divisible distributions, general state space Markov chains, ergodic theorems, large deviations, martingales, Brownian motion and Donsker's theorem. Prerequisites: Mathematics 241 or Statistics 205 or equivalent. Instructor: Staff. 3 units. C-L: Statistics and Decision Sciences 207

288. Topics in Probability Theory. Probability tools and theory, geared towards topics of current research interest. Possible additional prerequisites based on course content in a particular semester. Prerequisites: Mathematics 135 or equivalent, and consent of instructor. Instructor: Staff. 3 units. C-L: Statistics and Decision Sciences 297

295. Special Topics. Instructor: Staff. 3 units.

296. Special Topics. Instructor: Staff. 3 units.

298. Special Readings. Instructor: Staff. 3 units.

For Graduate Students Only

348. Current Research in Analysis. Not open to students who have taken Mathematics 388, 389. Instructor: Staff. 3 units.

358. Current Research in Algebra. Not open to students who have taken Mathematics 368-369. Instructor: Staff. 3 units.

368. Current Research in Topology. Not open to students who have taken Mathematics 378-379. Instructor: Staff. 3 units.

369. Current Research in Differential Geometry. Instructor: Staff. 3 units.

378. Research in Algebraic Geometry. Mini seminars on current topics which are repeatable for credit. Instructor: Staff. 1 unit.

379. Current Research in Mathematical Physics. Not open to students who have taken Mathematics 387. Instructor: Staff. 3 units.

388. Research in Differential Equations. Mini seminars on current topics which are repeatable for credit. Instructor: Staff. 1 unit.

389. Current Research in Applied Mathematics. Instructor: Staff. 3 units.

390. Teaching College Mathematics. This course is designed for first year mathematics graduate students as preparation for teaching as graduate students at Duke and as professors, once they graduate. Topics include lesson planning, overview of the content in calculus courses, current issues in undergraduate mathematics education, writing and grading tests, evaluating teaching and practice teaching. Consent of instructor required. Instructor: Staff. 1 unit.