

Duffing's Equation

Remark: Before proceeding, be sure to try out the introductory examples `ch1-riccati.ode` and `ch1-van-der-Pol.ode` and read their accompanying documentation. That won't take long and is well worth the effort! In what follows, we assume that you are familiar with the basics of XPP syntax and how to navigate some of its menus.

The plain text file `ch1-duffing.ode` is an XPP script for numerical solution of the Duffing equation

$$x'' + \beta x' - x + x^3 = 0,$$

where β is a parameter. As in the van der Pol example, start by writing Duffing's equation as a system of two first-order ODEs.

Here are a few experiments to try out with this XPP script:

1. Load the file `ch1-duffing.ode` into XPP by following the instructions at

<http://www.math.pitt.edu/~bard/xpp/ezwin.html>

You should see an empty plot with the viewing window set according to the default values set by `xlo`, `xhi`, `ylo`, `yhi` in the script file. Choose **I**nitialconds and then **G**o to solve the equation with the default initial conditions and parameter beta.

2. Mimic the steps we outlined in the instructions for the van der Pol equation example `ch1-van-der-Pol.ode` to create two slider bars: one that allows you to vary beta between 0 and 3, and another that lets you vary the initial condition for x between -1 to 1 .
3. Now experiment with these slider bars to explore how the parameter β and initial conditions affect the dynamics. What happens as you vary $x(0)$ from negative to positive values? As you vary beta, can you explain on physical grounds why that parameter has the observed effect on the solution?
4. To quit XPP, from the main menu select **F**ile, then **Q**uit, and finally **Y**es.
5. For more XPP documentation, be sure to refer to Bard Ermentrout's XPP website at

<http://www.math.pitt.edu/~bard/xpp/xpp.html>