Modified Lotka-Volterra Equations

Remark: Before proceeding, be sure to try out `ch1-riccati.ode` *and* `ch1-van-der-Pol.ode` *and read their accompanying documentation. That won't take long and is well worth the effort! In what follows, we assume that you are familiar with the basics of XPP syntax and how to navigate some of its menus.*

The plain text file `ch1-modified-LV.ode` is an XPP script for numerical solution of the modified Lotka-Volterra equations

$$\begin{aligned}\frac{dx}{dt} &= x\left(\frac{x-\epsilon}{x+\epsilon}\right)\left(1-\frac{x}{K}\right) - xy \\ \frac{dy}{dt} &= \rho(xy - y),\end{aligned}$$

where ϵ, K, and ρ are parameters. Refer to the textbook for a detailed description of these equations.

A word about some fairly self-explanatory syntax in the `ch1-modified-LV.ode` XPP script: We have defined a function

$$\phi(x) = \frac{x-\epsilon}{x+\epsilon}.$$

There are several advantages to doing so, one of which is that it makes it easier to read the right-hand side of the equation for dx/dt. It also makes it easier to experiment with other ways of simulating the Allee effect (i.e., by trying a different formula for $\phi(x)$).

Here are a few experiments to try out with this XPP script:

1. Load the file `ch1-modified-LV.ode` into XPP by following the instructions at

   `http://www.math.pitt.edu/~bard/xpp/ezwin.html`

   You should see an empty plot with the viewing window set according to the default values set by `xlo, xhi, ylo, yhi` in the script file. Choose **I**nitialconds and then **G**o to solve the equation with the default initial conditions and parameter values.

2. Mimic the steps we outlined in the instructions for the van der Pol equation example `ch1-van-der-Pol.ode` to create a slider bar that allows you to vary K between 0.1 and 7.1.

3. Experiment with the slider bar to explore how the parameter K affects the dynamics. You should observe that the system evolves to one of three steady-states (as described in the text) depending on the value of K: either (I) extinction of both species; (II) coexistence of both species; and (III) extinction of the predator species only.

4. In Section 1.6.2 of our textbook, Table 1.1 summarizes which of these three long-term behaviors is generic depending on the values of ϵ and K. Using our default value of $\epsilon = 0.1$, the Table indicates that the transition between behaviors I and II should occur at $K = 5.95$. *However, this is NOT what you will observe if you followed the above instructions—both species may go extinct for some* $K < 5.95$. So what's going on?

5. The inequalities in Table 1.1 (derived later in the textbook) are based upon analysis of the *local* behavior of solutions in the vicinity of the coexistence equilibrium. If you select initial conditions closer to that equilibrium (try it!), the transitions between behaviors I, II, and III occur at K values that are much closer to the ones predicted in Table 1.1.
6. Be sure to revisit this example as you proceed through the textbook, as it will be the subject of many instructive examples involving stability of equilibria, periodic orbits, and bifurcations!
7. To quit XPP, from the main menu select **F**ile, then **Q**uit, and finally **Y**es.
8. For more XPP documentation, be sure to refer to Bard Ermentrout's XPP website at

   `http://www.math.pitt.edu/~bard/xpp/xpp.html`