A Modified Pendulum Equation

Remark: Before proceeding, we recommend that you test out the introductory examples `ch1-riccati.ode` *and* `ch1-van-der-Pol.ode` *and read their accompanying documentation.*

The plain text file `ch1-pendulum.ode` is an XPP script for numerical solution of the modified pendulum equation

$$x'' + \beta x' + \left[1 + \alpha\omega^2 \cos(\omega t)\right] \sin x = 0,$$

where α, β, and ω are parameters and x measures the angle of the pendulum relative to the downward vertical position. In the special case $\alpha = \beta = 0$, the equation reduces to the standard pendulum equation, which is discussed in Section 1.4 of our textbook. In the more general equation above, $\beta x'$ is a friction term (higher β means increased friction) and the factor glued to $\sin x$ describes vertical vibration of the pendulum pin. The parameter α is proportional to the amplitude of the vertical vibration and the parameter ω sets the frequency of the vibrations (see Chapter 1 exercises in our textbook). As usual, this second-order ODE is written as a system of two first-order ODEs in the XPP script by introducing a new variable, $y = x'$.

Here are a few experiments to try out with this XPP script:

1. Load the file `ch1-pendulum.ode` into XPP. You should see an empty plot with the viewing window set according to the default values set by `xlo, xhi, ylo, yhi` in the script file.
2. To plot the solution of the ODE using the default initial conditions and parameters in `ch1-pendulum.ode`: From the menu, select **I**nitialconds and then choose **G**o from the submenu. A plot of the solution appears in the viewing window. Because this ODE is non-autonomous, we have elected to render a plot of x versus t here as opposed to a phase plot of y versus x.
3. Create a slider bar (see instructions from van der Pol equation example) that allows you to vary the parameter ω (the frequency of the vertical vibration of the pendulum pin).
4. Refer to the computational exercise at the end of Chapter 1 of our textbook: Start with the pendulum at rest in a nearly-vertical position, say $x(0) = 3.1$ (nearly π radians), and with parameters $\alpha = 0.1$ and $\beta = 0.05$. Gradually increase ω from 1. You should observe that if the pendulum pin is oscillated with low frequency, the pendulum eventually comes to rest in the downward vertical position (x approaches an integer multiple of 2π as $t \to \infty$). However, if ω is sufficiently large (determine how large!) it is possible to stabilize the pendulum in the *upward* vertical position (x approaches an odd-integer multiple of π as $t \to \infty$). What other types of long-term behaviors can you observe as ω is varied?
5. Feel free to create other slider bars to vary the other parameters and/or the initial conditions on x and y to see how these affect the dynamical behavior!
6. To quit XPP, from the main menu select **F**ile, then **Q**uit, and finally **Y**es.
7. For more XPP documentation, be sure to refer to Bard Ermentrout's XPP website at

   `http://www.math.pitt.edu/~bard/xpp/xpp.html`