Introductory example: Getting used to XPP

The plain text file `ch1-riccati.ode` is an XPP script for numerical solution of a Riccati equation

$$x' = x^2 - t.$$

There are several things to note about the syntax. First, lines beginning with the # symbol are merely comments. The first "real" input line is the declaration of the ODE itself, and syntax should be self-explanatory. In this case, XPP knows to interpret x as the dependent variable and t as the independent one. The `init` command is used to specify the default initial condition for this ODE, which we have set as $x(0) = 0.3$. The remaining commands set the default viewing window for visualizing solutions—note that those lines begin with the @ symbol. The `xplot` and `yplot` commands tell XPP which variables should correspond to the horizontal and vertical axes, respectively. The command `total` tells XPP how far forward in time to solve the ODE. In this case, `total = 10` indicates that the solution will be computed over the interval $0 \leq t \leq 10$. The `dt` command sets the step size that XPP will use when numerically approximating the solution of the ODE. If `dt` is chosen too small (e.g., to guarantee more precision), then you may need to tell XPP to reserve more memory to store all the data points using the `maxstor` command. More about this in future examples. Finally, the commands `xlo, xhi, ylo, yhi` set the default viewing window for the horizontal and vertical axes.

Quick Start: Plotting the solution of the initial value problem. Load the file `ch1-riccati.ode` into XPP by following the instructions[1] at

`http://www.math.pitt.edu/~bard/xpp/ezwin.html`

You should see an empty plot with the viewing window set according to the default values set by `xlo, xhi, ylo, yhi` in the script file. The label above the plot window indicates the variables that correspond to the two axes. The menu commands along the left side of the XPP window can be accessed either using the mouse or with the keyboard, using the keys corresponding to the capital letters in the various menu options. (Henceforth, we opt for the latter and will use **bold face** letters to indicate the keystrokes.)

1. To solve the ODE using the default initial condition in the `ch1-riccati.ode` file: From the menu, select **I**nitialconds. Then select **G**o. A trace of the solution appears in the viewing window.

2. The solution doesn't fit this viewing window particularly well, so from the main XPP window, you can select the command **W**indow and then choose **F**it to get a better view. The horizontal axis should stop at the value specified by `total` (see above).

3. If you'd like to see the solution further forward in time, choose the **C**ontinue command from the main menu. At the top of the screen, you can enter a new stopping time, say 15. Then, from the main menu select **W**indow and then **F**it to see the solution over the interval $0 \leq t \leq 15$.

[1] In this and all subsequent examples, we assume that you have installed and are running XPP under the Windows operating system with the help of an X-server program such as Xming. If you're running XPP under Linux or using a Mac or IPad, there will be a few differences (e.g., in how you open files) and we leave it to you to make the minor adaptations to the instructions that follow.

4. Next, let's explore how the initial condition affects the behavior of the solution. First, use the **E**rase command to clear the viewing window and then **W**indow and **D**efault to restore the default viewing window. To solve the ODE using a range of initial conditions, under **I**nitialconds select **R**ange. A new window pops up, and its first four fields prompt you for the following information: (i) the dependent variable whose initial condition you wish to vary (in this case, x is the only possible choice); (ii) the number of different [equally spaced] initial conditions to use; (iii) the smallest initial condition to use; and (iv) the largest initial condition to use. In those four fields, enter x, 20, 0 and 1, respectively and then click **OK**. You should see 20 different solutions of the ODE, for initial conditions $x(0)$ varying from $x(0) = 0$ to $x(0) = 1$. Notice that there seems to be some critical value of $x(0)$ below which solutions ultimately decay without bound and above which solutions ultimately grow without bound.

5. Fun with graphics: **E**rase the window and again navigate to **I**nitialconds and then **R**ange. In the "Cycle Color" option, choose "Y" if you'd like to use different colors for different initial conditions. Alternatively, under "Movie", select "Y" if you'd like to see an animation that show how the solution trajectory changes as the initial condition varies over the range you selected.

6. Remark: You can also adjust the initial condition using the "ICs" button at the top of the XPP window, or from the main menu by selecting **I**nitialconds and then **N**ew. If you choose the latter option, you'll be prompted to enter a new choice for $x(0)$ at the top of the screen.

7. Once you're accustomed to the commands described above, try out some of the other XPP scripts involving systems of ODEs and/or ODEs containing parameters.

8. To quit XPP, from the main menu select **F**ile, then **Q**uit, and finally **Y**es.

9. For more XPP documentation, be sure to refer to Bard Ermentrout's XPP website at

`http://www.math.pitt.edu/~bard/xpp/xpp.html`