The van der Pol Equation

Remark: Before proceeding, we recommend that you test out the introductory example `ch1-riccati.ode` *and read its accompanying documentation.*

The plain text file `ch1-van-der-Pol.ode` is an XPP script for numerical solution of the van der Pol equation

$$x'' + \beta(x^2 - 1)x' + x = 0,$$

where β is a parameter. For ODEs that are second-order or higher, you'll need to write the ODE as a system of first-order ODEs when entering them into XPP. In this case, set $y = x'$ and write van der Pol's equation as the system

$$\begin{aligned} \frac{dx}{dt} &= y \\ \frac{dy}{dt} &= -\beta(x^2 - 1)y - x. \end{aligned}$$

A few quick remarks regarding syntax in the file `ch1-van-der-Pol.ode` that did not occur in our introductory example with the Riccati equation. First, the command `param beta=1.0` simultaneously declares that β is to be treated as a parameter and sets its default value to be 1.0. *Do not put spaces between the name of the parameter, the equal sign, and the value of the parameter!* Second, note that asterisks are needed to indicate multiplication on the right-hand side of the ODE for $\frac{dy}{dt}$, and XPP will issue an error if those asterisks are omitted. This time, a pair of default initial conditions is specified using the `init` command, one for each dependent variable. Finally, the default viewing window is set up to display a phase plane plot.

Here are a few experiments to try out with this XPP script:

1. Load the file `ch1-van-der-Pol.ode` into XPP by following the instructions at

   `http://www.math.pitt.edu/~bard/xpp/ezwin.html`

   You should see an empty plot with the viewing window set according to the default values set by `xlo, xhi, ylo, yhi` in the script file.

2. To solve the ODE using the default initial conditions and parameter β in the file `ch1-van-der-Pol.ode`: From the menu, select **I**nitialconds. Then select **G**o. A solution trajectory, parametrized by t, appears in the viewing window.

3. To see a plot of x versus t instead, from the main menu choose **X**i vs t. At the top of the screen, you will be prompted to select a variable to plot versus t. Type x and press enter. Generating a plot of y versus t is similar.

4. To return to the y versus x phase plane plot, from the main menu select **V**iewaxes and choose **2**D. In the "X-axis" and "Y-axis" fields, enter x and y. You can specify the viewing window if you like, or just click "OK". From the main menu, select **W**indow, then **F**it to restore a convenient viewing window.

5. There are several ways to change the parameter β. If you have a *specific* value of β you wish to try, select **P**arameters from the main menu, enter "beta" at the top of the screen, type the new value of beta, and then hitting Escape. Finally, choose

Initialconds and **G**o. Alternatively, you can click the "Param" button at the top of the XPP window, enter a new value of beta in the box and click "Go" to see an updated solution trajectory.

6. A far better way to explore how a parameters affect the solution of an ODE is to create slider bars. Click one of the fields labeled "Par/Var?" at the bottom of the XPP window. A new window pops up prompting you to create a slider bar. In the first field, enter "beta". In the other three fields, enter the starting value of beta, along with the lowest and highest values of beta that you would like to include in the slider bar, then click "OK". Try this with the numbers 1.0, 0.0, and 2.0, respectively. A slider bar is now created allowing you to vary beta. **E**rase any existing plot and set the window to its default view (**W**indow/zoom and then **D**efault). Click the "Go" button on the slider window you created. In order to vary beta between its low and high values, click and drag the slider bar from side-to-side. The solution of the ODE is updated in real time.

7. You can even create a slider bar to vary an *initial condition*. Click on one of the other fields labeled "Par/Var?" at the bottom of the XPP window. In the window that pops up, enter x (a dependent variable) in the first field. In the other three fields, input the default initial condition $x(0)$ followed by the lowest and highest values of that initial condition to use on the slider bar. Click "OK" when finished, and you should now have a new slider bar that lets you vary the initial condition on x.

8. Be sure to experiment with these slider bars to explore how the parameter and initial conditions affect the dynamics!

9. To quit XPP, from the main menu select **F**ile, then **Q**uit, and finally **Y**es.

10. For more XPP documentation, be sure to refer to Bard Ermentrout's XPP website at

    `http://www.math.pitt.edu/~bard/xpp/xpp.html`