An Activator-Inhibitor System

Remark: Before proceeding, we recommend that you familiarize yourself with basic XPP syntax via the introductory Chapter 1 examples `ch1-riccati.ode` *and* `ch1-van-der-Pol.ode` *and their accompanying documentation.*

The plain text file `ch4-activator-inhibitor.ode` is an XPP script for numerical solution of the equations

$$\begin{aligned} x' &= \frac{\sigma}{1+y}\,\frac{x^2}{1+x^2/\kappa^2} - x \\ y' &= \rho\left[\frac{x^2}{1+x^2/\kappa^2} - y\right], \end{aligned}$$

which models an activator-inhibitor system (See Section 4.4.2 of our textbook). The parameters σ, ρ, and κ are all positive.

The default parameter values, initial conditions, and viewing window are all specified in the `ch4-activator-inhibitor.ode` file. For the purposes of the following exercises, the default viewing window and parameter values serve as a useful starting point.

Here are some experiments to try with this XPP script:

1. Load the file `ch4-activator-inhibitor.ode` into XPP. Use **I**nitialconds and **G**o to plot a solution trajectory (in the y versus x phase plane) using the default initial conditions and parameter choices.

2. Without erasing the trajectory that appeared in the preceding step, create a slider bar that allows you to vary the parameter σ between 1 and 4, and set $\sigma = 4$ to start with. See what happens when you gradually reduce σ. You should notice a major change in the long-term behavior of the solution once σ crosses below some threshold value.

3. To better understand the behavior you observed in the previous step, let's ask XPP to plot the *nullclines* for this activator-inhibitor system. From the main XPP menu, select **N**ullclines and then choose **N**ew. Now use the slider bar to vary σ again. The viewing window should now display not only the solution trajectory, but also the nullclines for this system of ODEs. As you vary σ, notice the effect that parameter has on the nullclines.

4. You may wish to create a second slider bar to vary the parameter κ as well. (The final parameter ρ has no effect on the nullclines, although it does alter the solution trajectories.)

5. The initial conditions also play an important role in the long-term behavior of solutions. Set your slider bar(s) so that $\sigma = 4$, $\kappa = 1$, and $\rho = 0.5$, and use the default viewing window. In order to plot a phase portrait with lots of representative trajectories, from the main menu select **D**ir. field/flow and then **F**low. At the top of the screen, you will be prompted to enter a number that tells XPP how fine of a "Grid" to sample from when selecting initial conditions for the trajectories that will be plotted (the default value is 10). Either hit `Enter` to accept the default value or consider changing the 10 to a 5 in order to reduce the number of trajectories that are plotted. In the resulting phase portrait, you should see that depending upon initial conditions, trajectories generically approach one of two equilibria.

6. Feel free to play around with parameters to get a sense of how they affect the generic long-term behavior of solutions.

7. For more XPP documentation, be sure to refer to Bard Ermentrout's XPP website at

`http://www.math.pitt.edu/~bard/xpp/xpp.html`