Chemostat Model

Remark: Before proceeding, we recommend that you familiarize yourself with basic XPP syntax via the introductory Chapter 1 examples `ch1-riccati.ode` *and* `ch1-van-der-Pol.ode` *and their accompanying documentation.*

The plain text file `ch4-chemostat.ode` is an XPP script for numerical solution of the equations

$$\begin{aligned} x' &= \frac{xy}{y+1} - \rho x \\ y' &= -\frac{xy}{y+1} - \rho(y-\sigma), \end{aligned}$$

where ρ and σ are positive parameters (See Sections 4.3.2 and 5.5.2 of our textbook for details). If the inequalities $\rho < 1$ and $\sigma > \rho/(1-\rho)$ are satisfied, then there is an equilibrium in the (open) first quadrant.

The default parameter values, initial conditions, and viewing window are all specified in the `ch4-chemostat.ode` file. For the purposes of the following exercises, the default viewing window and parameter values serve as a useful starting point.

Here are some experiments to try with this XPP script:

1. Load the file `ch4-chemostat.ode` into XPP. Use **I**nitialconds and **G**o to plot a solution trajectory (in the y versus x phase plane) using the default initial conditions and parameter choices.
2. Have XPP plot the *nullclines*: From the main XPP menu, select **N**ullclines and then choose **N**ew.
3. Keeping $\rho = 0.5$ fixed, create a slider bar allowing σ to be varied between 0.5 and 2.5. Notice that the y-nullcline moves as σ is varied.
4. Equilibria occur where an x-nullcline crosses a y-nullcline. Where are the equilibria if $\rho = 0.5$ and $\sigma = 2$? What about if $\rho = 0.5$ and $\sigma = 0.5$?
5. To sketch a phase portrait one trajectory at a time, you may enter the commands **I**nitialconds, m**(I)**ce and repeatedly click your mouse button at various locations within the viewing window. The coordinates you click on are used as initial conditions for trajectories. Hit the escape key `Esc` when you wish to stop clicking. Try this out with $\rho = 0.5$ and $\sigma = 2$. Generically, what is the long-term behavior of the solutions?
6. From the main menu, use the **E**rase command to clear the window, and change σ to 0.5 while keeping $\rho = 0.5$. As before, use **I**nitialconds, m**(I)**ce and click your mouse on various locations within the viewing window. Now what is the generic behavior of solutions?
7. If you performed the preceding calculations correctly, you should have noticed that for $\sigma = 2$, trajectories [generically] approach an equilibrium for which both coordinates are positive, while if $\sigma = 0.5$, trajectories [generically] approach an equilibrium for which $x = 0$.
8. For more XPP documentation, be sure to refer to Bard Ermentrout's XPP website at

   `http://www.math.pitt.edu/~bard/xpp/xpp.html`