# Sel'kov's Model for Glycolysis

*Remark: Before proceeding, we recommend that you familiarize yourself with basic XPP syntax via the introductory Chapter 1 examples* `ch1-riccati.ode` *and* `ch1-van-der-Pol.ode` *and their accompanying documentation.*

The plain text file `ch4-selkov.ode` is an XPP script for numerical solution of the equations

$$\begin{aligned} x' &= \rho - \sigma x - xy^2 \\ y' &= -y + \sigma x + xy^2, \end{aligned}$$

where $\rho$ and $\sigma$ are positive parameters (See Section 4.4.3 of our textbook for details).

The default parameter values, initial conditions, and viewing window are all specified in the `ch4-selkov.ode` file. For the purposes of the following exercises, the default viewing window and parameter values serve as a useful starting point.

Here are some experiments to try with this XPP script:

1. Load the file `ch4-selkov.ode` into XPP. Use **I**nitialconds and **G**o to plot a solution trajectory (in the $y$ versus $x$ phase plane) using the default initial conditions and parameter choices.

2. Let's ask XPP to plot the *nullclines*: From the main XPP menu, select **N**ullclines and then choose **N**ew.

3. Create a slider bar that allows you to vary the parameter $\sigma$ from 0.05 to 0.2, and choose $\sigma = 0.2$ to start with. You may create a slider bar for $\rho$ if you wish, but keep $\rho = 0.75$ for now.

4. Using the slider bar to reduce $\sigma$ gradually from 0.2 to 0.1 and observe how the nullclines change. The solution trajectory also experiences an interesting change in its long-term behavior: instead of spiraling inward towards an equilibrium, something very different happens.

5. With $\sigma = 0.1$ and $\rho = 0.75$, you may wish to repeatedly hit **I**nitialconditions, **L**ast. (Choosing **L**ast tells XPP to continue the most recently computed solution forward in time.) Here, you should observe that the solution trajectory approaches a closed orbit.

6. If you would like to see a phase portrait for these equations, from the main menu select **D**ir. field/flow and then **F**low. At the top of the screen, you will be prompted to enter a number that tells XPP how fine of a "Grid" to sample from when selecting initial conditions for the trajectories that will be plotted (the default value is 10). Either hit `Enter` to accept the default value or consider changing the 10 to a 5 in order to reduce the number of trajectories that are plotted.

7. Although it may not be completely apparent from the phase portrait, for $\sigma = 0.1$ and $\rho = 0.75$ there is precisely *one* closed orbit, and trajectories starting near the closed orbit spiral towards it as $t \to \infty$. Isolated periodic orbits like this one are known as *limit cycles* and are studied in Chapter 7.

8. Feel free to play around with parameters to get a sense of how they affect the generic long-term behavior of solutions.

9. For more XPP documentation, be sure to refer to Bard Ermentrout's XPP website at

`http://www.math.pitt.edu/~bard/xpp/xpp.html`