Turing Instability

Remark: Before proceeding, we recommend that you familiarize yourself with basic XPP syntax via the introductory Chapter 1 examples `ch1-riccati.ode` and `ch1-van-der-Pol.ode` and their accompanying documentation.

The plain text file `ch6-turing.ode` is an XPP script for numerical solution of the equations

$$\begin{aligned} x_1' &= \sigma x_1^2/(1+y_1) - x_1 \\ y_1' &= \rho[x_1^2 - y_1] + D(y_2 - y_1) \\ x_2' &= \sigma x_2^2/(1+y_2) - x_2 \\ y_2' &= \rho[x_2^2 - y_2] + D(y_1 - y_2) \end{aligned}$$

where ρ, σ and D are positive parameters (See Sections 6.3.2 and 8.6.1 of our textbook for details).

The default parameter values, initial conditions, and viewing window are all specified in the `ch6-turing.ode` file. For the purposes of the following exercises, the default viewing window and parameter values serve as a useful starting point.

Here are some experiments to try with this XPP script:

1. Load the file `ch6-turing.ode` into XPP. Use **I**nitialconds and **G**o to plot a solution trajectory (projected on the x_2 versus x_1 phase plane) using the default initial conditions and parameter choices.
2. Create a slider bar that allows you to vary the diffusion coefficient parameter D from 0 to 1, with a starting value of $D = 0.1$. For the other parameters, use $\sigma = \rho = 2.5$. For small D, the "trivial" equilibrium $(x_1, y_1, x_2, y_2) = (2, 4, 2, 4)$ is locally stable. You should observe that the solution trajectory approaches that equilibrium if D is small.
3. Use the slider bar to gradually increase D. Once D is appropriately large, the aforementioned equilibrium loses stability and nearby trajectories approach very different equilibrium states as $t \to \infty$. You may wish to convince yourself of this by choosing initial conditions very close to the "trivial" equilibrium and observing what happens as D is gradually increased.
4. Feel free to play around with parameters to get a sense of how they affect the generic long-term behavior of solutions.
5. For more XPP documentation, be sure to refer to Bard Ermentrout's XPP website at

   `http://www.math.pitt.edu/~bard/xpp/xpp.html`