MATH 340 – SPRING 2026 – ADDITIONAL PROBLEMS TO PRACTICE FOR THE FIRST MIDTERM

Not to be turned in. Please don't feel compelled to solve all of them!

Problem 1. Here are some good exercises from Meester: 1.7.15, 1.7.35, 1.7.36, 2.7.11, 2.7.15, 2.7.17, 2.7.18, 2.7.28, 2.7.29, 2.7.30, 2.7.32, 2.7.33, 2.7.38, 4.3.1, 4.3.2. Many of the other problems from the sections we have covered in Meester are good, too.

Problem 2. Let X be a random variable that takes on values between 0 and c. That is, $P[0 \leq X \leq c] = 1$. Show that $\mathrm{Var}(X) \leq c^2/4$. [*Hint*: first argue that $\mathbb{E}[X^2] \leq c\mathbb{E}[X]$, and then use this inequality to show that $\mathrm{Var}(X) \leq c^2[\alpha(1-\alpha)]$, where $\alpha = \mathbb{E}[X]/c$.]

Problem 3. Let X be a random variable such that $E[X^2] < \infty$. Show that

$$P(X > 0) \geq \frac{(E[X])^2}{E[X^2]}.$$

[*Hint*: start by writing $E[X] = E[X\mathbf{1}_{X>0}]$.]

Problem 4. In a single-elimination tournament with 2^n players, in which the players are ranked by strength and a stronger player always defeats a weaker one, but in which the seedings are decided uniformly at random, what is the probability that the second-strongest player loses to the strongest in the finals?

Problem 5. In problem of the chaotic airplane passengers of Homework 1, Problem 3, the first k of the n seats are designated as first class, and the remaining $n-k$ seats are designated as economy.

(a) What the expected number of economy passengers who end up sitting in first class?
(b) What is the variance of the number of economy passengers who end up sitting in first class?

Problem 6. Let X and Y be random variables on the same probability space. Let

$$\mathrm{Var}(X \mid Y) = E[X^2 \mid Y] - (E[X \mid Y])^2.$$

Prove that

$$\mathrm{Var}(X) = \mathrm{Var}(E[X \mid Y]) + E[\mathrm{Var}(X \mid Y)].$$

Problem 7. For the zeta distribution discussed in Homework 3, Problem 6, for which values of s is the expectation finite? What about the variance?