MATH 541, FALL 2023, HOMEWORK 2

Due Saturday, September 23, 2023 at 5pm Eastern time on Gradescope. Write solutions to exercises 2.2, 2.3, 2.4, and 2.5. In addition, solve the following two exercises:

Problem 1. Lawler, exercise 2.2.

Problem 2. Lawler, exercise 2.3.

Problem 3. Lawler, exercise 2.4.

Problem 4. Lawler, exercise 2.5

Problem 5. Let $X_0, X_1, X_2, \ldots$ be a simple random walk on $\mathbb{Z}$, with $X_0 = 0$. Let $T = \min\{n : X_n = 1\}$.

   (a) For $\lambda < 0$, derive a formula for $\mathbb{E}e^{\lambda T}$. [*Hint*: condition on the first step the chain takes.] Is the resulting expression differentiable at $\lambda = 0$?

   (b) Use a computer to simulate 100, 1000, and 10000 independent realizations of T. Plot a histogram of your results in each case.