MATH 541, FALL 2023, HOMEWORK 6

Due Friday, October 20, 2023 at 5pm Eastern time on Gradescope. Solve the following exercises:

Problem 1. Lawler, exercise 7.2.

Problem 2. Lawler, exercise 7.9.

Problem 3. Code up the Metropolis algorithm for the Ising model on 30×30 matrices. Create a simulation for the Ising model on 50×50 matrices with entries in ± 1. Start the chain with initial data that is uniform among all matrices with ± 1 entries (i.e. each entry is iid Rademacher), and run it for 100000 steps. Draw the final value of the chain (choose one color for $+1$ and another color for -1) for (a) $\beta = 0$, (b) $\beta = 0.05$, (c) $\beta = 0.1$, (d) $\beta = 0.15$, (e) $\beta = 0.2$, and (f) $\beta = 0.5$. You should notice qualitative difference between the plots: write a brief interpretation. Also include your code in your submission.