EXAM 3

Math 219, 2023 Summer Term 1, Clark Bray.

Name:____ Section:______ Student ID:____________

GENERAL RULES

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

No notes, no books, no calculators.

All answers must be reasonably simplified.

All of the policies and guidelines on the class webpages are in effect on this exam.

WRITING RULES

Do not remove the staple, tear pages out of the staple, or tamper with the exam packet in any way. Do not write anything near the staple – this may be cut off.

Use black pen only. You may use a pencil for initial sketches of diagrams, but the final sketch must be drawn over in black pen and you must wipe all erasure residue from the paper.

Work for a given question can be done ONLY on the front or back of the page the question is written on. Room for scratch work is available on the back of this cover page, and on the two blank pages at the end of this packet; scratch work will NOT be graded.

DUKE COMMUNITY STANDARD STATEMENT

"I have adhered to the Duke Community Standard in completing this examination."

Signature: __________________________

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1. *(20 pts)* S has equation $z = x^2 + y^2$, and C is the shortest curve on S that starts at $(1, 0, 1)$ and ends at $(0, 2, 4)$. Compute the line integral over C of $\vec{F}(x, y, z) = (2xyz, x^2z, x^2y + 3z^2)$.

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2. *(20 pts)* The xy-plane consists of 1×1 "tiles" with vertices at the points with integer coordinates. The region R consists of all of those tiles that touch the solid disk $x^2 + y^2 = 7$. Compute the circulation around the boundary of R of the vector field $\vec{G}(x, y) = (x - 2y, 3x - 4y)$.

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3. *(20 pts)* S is the part of the plane $3x - 2y + 6z = 7$ sitting above the unit square in the xy-plane, oriented downward. Compute the flux through S of $\vec{F}(x, y, z) = (4y, 3x, 6z)$.

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4. *(20 pts)*

(a) Air velocity is described by $\vec{F}(x, y, z) = (x + z, 3x + 2y, y - 3z)$. The air is causing a small balloon at the point $(2, 1, 3)$ to spin. Find a vector parallel to the axis around which the ballon is spinning.

(b) R is a rectangular box surface with all six faces parallel to the coordinate planes, and opposite vertices at $(0, 0, 0)$ and $(2, 3, 4)$. The orientation of R at $(1, 1, 0)$ is upward. Compute the flux through R of the vector field $\vec{G}(x, y, z) = (xy, 3y - 5z, 4 - yz)$.

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5. *(20 pts)* The surface S has equation $x^2 + (y-3)^2 + z^2 = 1$, oriented inward. H is the part of S with $x \leq 0$. Compute the flux through H of $\vec{F}(x, y, z) = (-y, x, 0)$.

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