EXAM 1

Math 219, 2023 Fall, Clark Bray.

Name:_____ Section:_____ Student ID:_____

GENERAL RULES

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

No notes, no books, no calculators.

All answers must be reasonably simplified.

All of the policies and guidelines on the class webpages are in effect on this exam.

WRITING RULES

Do not remove the staple, tear pages out of the staple, or tamper with the exam packet in any way. Do not write anything near the staple – this may be cut off.

Use black pen only. You may use a pencil for initial sketches of diagrams, but the final sketch must be drawn over in black pen and you must wipe all erasure residue from the paper.

Work for a given question can be done ONLY on the front or back of the page the question is written on. Room for scratch work is available on the back of this cover page, and on the two blank pages at the end of this packet; scratch work will NOT be graded.

DUKE COMMUNITY STANDARD STATEMENT

"I have adhered to the Duke Community Standard in completing this examination."

Signature: _____

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1. *(20 pts)*

(a) The line L in $\mathbb{R}^3$ contains the points $(1, 0, 0)$ and $(3, 4, 2)$. The surface S has equation $x^2 + y - z^2 = 17$ (notice that the y in this equation is not squared!). Find all points of intersection of L and S.

(b) The linear transformation T gives $T(1, 0, 0) = (-1, 0, 0)$, $T(0, 1, 0) = (0, 0, 1)$, $T(0, 0, 1) = (0, 1, 0)$. Find a parametrization of the line M that results from applying T to L (from part (a)).

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2. *(20 pts)* In this question we have $\vec{a} = (1, 4, 2)$, $\vec{b} = (2, 1, 0)$, $\vec{c} = (0, 1, 1)$.

(a) Find the projection of $\vec{a}$ in the direction of $\vec{c}$.

(b) Decide if the list $(\vec{a}, \vec{b}, \vec{c})$ is right handed or left handed.

(c) Compute the area of the parallelogram defined by $\vec{b}$ and $\vec{c}$.

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3. *(20 pts)*

(a) The point $\vec{a} = (3, 2, 1)$ is the point on the plane P in $\mathbb{R}^3$ that is closest to $\vec{b} = (4, 5, 6)$. Find the equation of the plane P.

(b) Suppose $f(x, y) = y - \sin x$. Find the equation for the level set $f^{-1}(3)$, and find a function g (indicate the domain, the target, and the formula) whose graph is that level set.

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4. *(20 pts)*

(a) A curve C (in the yz-plane) is rotated around an axis L in that plane to generate a surface R, which is then stretched by a factor of a in the x-direction and by a factor of b in the y-direction. The result is the surface S with equation $z = 4x^2 + 9y^2$. Find a, b, L, and the equation for C.

(b) We have $f : \mathbb{R}^2 \to \mathbb{R}^1$ is differentiable, with $\frac{\partial f}{\partial x} = 2$ and $\frac{\partial f}{\partial y} = 3$.

i. Is the graph steeper in the x-direction or the y-direction?

ii. Are the level sets farther apart in the x-direction or the y-direction?

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5. *(20 pts)* We consider here the differentiable function $f(x, y) = (xy, x^2 - y^3)$ and the point $\vec{a} = (2, 1)$.

   (a) Find the linear approximation of f at $\vec{a}$.

   (b) A particle $\vec{x}$ is at $\vec{a}$ and moving with instantaneous velocity $\dfrac{d\vec{x}}{dt} = (1, 2)$. Find the velocity $\dfrac{df(\vec{x})}{dt}$ of its image.

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