EXAM 2

Math 219, 2024 Fall, Clark Bray.

Name:____ Section:______ Student ID:____________

GENERAL RULES

YOU MUST SHOW ALL WORK AND EXPLAIN ALL REASONING TO RECEIVE CREDIT. CLARITY WILL BE CONSIDERED IN GRADING.

No notes, no books, no calculators.

All answers must be reasonably simplified.

All of the policies and guidelines on the class and course websites are in effect on this exam.

WRITING RULES

Do not remove the staple, tear pages out of the staple, fold or "dog-ear" pages, or tamper with the exam packet in any way. Do not write anything near the staple – this may be cut off.

Use black pen only. You may use a pencil for initial sketches of diagrams (only!), but the final sketch must be drawn over in black pen and you must wipe all erasure residue from the paper.

Work for a given question can be done ONLY on the front or back of the physical page the question is written on. Room for scratch work is available on the back of this cover page, and on the two blank pages at the end of this packet; scratch work will NOT be graded.

You may use a straight edge to assist in your drawings, but ONLY if there is zero mathematical content on the item.

DUKE COMMUNITY STANDARD STATEMENT

"I have adhered to the Duke Community Standard in completing this examination."

Signature: ________________________

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1. *(20 pts)*

(a) Compute the rate of change with respect to distance traveled of $f(x, y) = x^2 - y^2$ at the point $(5, 3)$ in the direction indicated by the vector $(5, 12)$.

(b) Find a vector orthogonal to the surface $x^3y - y^3z = 1 - xyz$ at the point $(1, 1, 2)$.

(c) Compute the flux of the vector field $\vec{F}(x, y, z) = (3, 1, 2)$ through the part of the plane $x + 2y + 2z = 9$ that is inside the sphere $x^2 + y^2 + z^2 = 25$. *(Hint: The distance from the origin to the above plane is 3.)*

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2. *(20 pts)* The domain D is defined by the equations $4x^2 + y^2 \leq 25$ and $(x-6)^2 + y^2 \leq 25$.

   (a) Set up completely (but do not evaluate!) an iterated double integral (just one!) representing $\iint_D ye^x \, dA$.

   (b) Compute the given double integral above by the most efficient possible method.

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3. *(20 pts)* The solid R is bounded by the planes $x = 0$, $y = 0$, $x + y = 1$, $x + y + z = 3$, and $x + y + z = -2$. Compute $\iiint_R x\, dV$.

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4. *(20 pts)* The curve $C_1 \in \mathbb{R}^2$ is the result of rotating $\{y = \sin x, x \in [0, \pi]\}$ by an angle $\pi/3$ counterclockwise around the origin, and the curve $C_2 \in \mathbb{R}^2$ is the result of rotating $\{y = -\sin x, x \in [0, \pi]\}$ by an angle $\pi/3$ counterclockwise around the origin. The region D is bounded by these two curves.

   Compute the integral $\iint_D y\, dx\, dy$ by using a change of variables. (You may stop computing when you have the resulting iterated integral reduced to a single variable integral.)

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5. *(20 pts)* B_1 and B_2 are balls centered at the origin, with radii 1 and 2, respectively. Let R be the spherical shell consisting of points that are in B_2 but NOT in B_1. Compute the integral below.

$$\iiint_R \frac{e^{x^2+y^2+z^2}}{\sqrt{x^2+y^2+z^2}}\,dx\,dy\,dz$$

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