Math 627 Homework #2, Fall 2022

Instructor: Ezra Miller

Solutions by: ...your name...

Collaborators: ...list those with whom you worked on this assignment...

Due: Tuesday 27 September 2022

READING ASSIGNMENTS in [Vakil]

- by Tuesday 20 September: Chapter 3; most of this should be review
- by Thursday 22 September: §4.1–§4.4
- by Tuesday 27 September: §4.5, Chapter 5 (much of this should be review)
- by Thursday 29 September: §11.1–§11.3, §12.1–§12.3 and §12.5; this plus Chapter 5 is a lot of material, but most should be review (skip items mentioning scheme morphisms)

EXERCISES: In [Vakil], exercises have labels C.S.N, for "Chapter C, Section S, Exercise N", where $C, S \in \mathbb{Z}_+$ and $N \in A, \ldots, Z$. Exercises marked "[essential]" are essential.

2.6.J

2.7.C

2.7.G (a)
(b) [essential]
(c) [essential]

2.5.D [essential]

3.2.Q [essential]

3.4.C (a)
(b)
(c)

3.4.H [essential]

3.6.L

3.6.F (a)
(b)

3.6.T

3.7.F [essential]

3.6.Q

3.7.E

4.1.A

4.3.F [essential]

13.1.A

13.1.C [essential]

13.1.E [essential]

Non-book Exercise

1. Fix a coherent sheaf $\mathcal{F}$ on a scheme $(X, \mathcal{O}_X)$. Prove that the set of points $\mathfrak{p} \in X$ where the fiber $\mathcal{F}(\mathfrak{p})$ has dimension at least r is closed in X, for each $r \geq 0$. Hint: what condition on an $m \times n$ matrix with entries in a field guarantees that it has rank at most $n - r$? [You need only what we did in class concerning coherent sheaves for this.]

References

[Vakil] Ravi Vakil, *The Rising Sea: Foundations of Algebraic Geometry*, November 18, 2017