

Index

- 3-connected, *53**
- abelian group, 129, 149
cyclic, 194
divisible, *see* divisible group
finite, 172, 194
finitely generated, 129
free, 131, 133
sequence defines multigrading, 149, 191
torsion, 152, 161
torsion-free, 151, 152, 187
- acyclic cover, *94*
- additive identity, 129
- additivity, *166*, 169, 172, 306, 311
yields Schubert polynomials, 323–324
- adjacent transposition, *291*, *298*, 303, 325
- adjointness, *see* functor, adjoint
- admissible family, *374*
universal, *374*
- admissible ideal, *373*, *374*, *377*
- Alexander duality, 81, 105–106
as planar map duality, 99–100, 106
on antidiagonal ideals, 318
on arbitrary ideals, *88*, 89–91, 226, 269
on cogeneric ideals, 123
on free and injective resolutions, 106
on free and irreducible resolutions, 225
on free resolutions, *see* duality for resolutions
on generic ideals, 122
on homological invariants, 100, 102–104
on irrelevant ideal, 199
on \mathbb{N}^n -graded modules, *228*
on upper bound problems, 125
principle behind, 96, 126
simplicial, *16*, 17, 81, 85, 98, 105
squarefree, *16*, 81–82, 89, 102, 226, 318
tight, *104*
topological, 83, 84
- Alexander inversion formula, 86, 106
- algebraic geometry, viii, 21, 41, 106, 193, 353, 355, 376
- algebraic shifting, 40, 45, 106
- algebraic torus, 21, 172, 191, 197, 200, 363
coordinates on, 192
- almost 3-connected, *53*, 54
- almost n -connected, *59*
- antidiagonal complex, *318*, 319–323, 329
from matrix Schubert variety, 323
is ball or sphere, 329
is shellable, 327
is subword complex, 327
- antidiagonal term, *280*, *318*
caused by rank condition, *321*
- associated prime
multigraded, 133, 152, 166
of Borel-fixed ideal, 39
of initial ideal, 145
of local cohomology, 254, 256, 270
of principal ideal, 147, 269
of \mathbb{Z} -graded module, 263
- Auslander–Buchsbaum formula, 100, 264
- ball, 145, 329, 330
- Barvinok’s algorithm, 229, 244
- Barvinok’s Theorem, 241
- barycenter, *112*
- basis weights, *158*
- Bass number, 104, 106, *223*, 224, 228, 265
of local cohomology, 255, 270
- Bayer, Dave, 86, 106
- Bender–Knuth involution, 329
- bet**ti diagram, *102*, 103
- Betti number, *see also* syzygy
characteristic dependence, 18, 58, 80
dual to Bass number, 104
duality for, 76, 98
extremal, *see* extremal Betti number
from cellular resolution, 65–66
multigraded, *157*
 \mathbb{N}^n -graded, *14*, 15–18
of Borel-fixed ideal, 30–33, 38
of generic ideal, 53, 112
of generic lattice ideal, 190
of lattice ideal, 174, 175
of lex-segment ideal, 35
of local cohomology, 255
of monomial ideal, 16, 85

*Italic page numbers refer to definitions

- of Stanley–Reisner ideal, 17, 85
- of trivariate ideal, 53
- of twisted cubic, 174
- under deformation, 119
- upper bound on, 53, 119–121
- upper-semicontinuity, 160
- Białynicki-Birula decomposition, 363
- Bigatti–Hulett Theorem, 35, 39
- binary complexity model, 241
- Binet–Cauchy formula, 339
- binomial ideal, *see* lattice ideal
- bit size, 241
- blowup, 197, 206, 372
- Borel group, 21, 299, 342, 346
- Borel–Weil Theorem, 288
- Borel-fixed ideal
 - advantage of, 41
 - generic initial ideal is, 24, 26, 35, 38
 - in positive characteristic, 40
 - variable-swapping characterizes, 23
- boundary (chain), 9, 62
- boundary map, *see* (co)boundary map
- braid relations, 305, 308
- Bridgeland–King–Reid Theorem, 368
- Brion’s Formula, 229, 237–243, 246
- Bruhat order, 295, 298
 - characterization by ranks, 296
 - characterization by row switch, 297
 - on symmetric group, 309
 - rank function on, 308
 - respects length, 302
- Buchberger graph, 48, 60
 - characterization of genericity by, 109
 - embedded in staircase, 60
 - is almost n -connected, 59
 - of generic ideal, 50, 111
 - planarity, 49–50, 58, 59, 75
 - Scarf edges lie in, 110, 125
- Buchberger map, 51, 59
- Buchberger’s Criterion, 47, 359, 370
- Buchberger’s Second Criterion, 48

- canonical module, 263, 265–266
 - of normal semigroup ring, 235
 - of polynomial ring, 254
- of semigroup ring, 233, 236, 267
- Carathéodory property, 141
- categorical quotient, 203, 204
- Čech complex, 250, 251, 253, 260
 - canonical: $\check{C}_{\Delta}^{\bullet}$, 259, 262, 270
 - from Taylor resolution, 260
 - generalized: $\check{C}_{\mathcal{F}}^{\bullet}$, 259, 260, 261
- Čech hull, 260, 261, 269, 270
- cell complex, 62, 77, *see also* simplicial complex
 - acyclic, 64, 66, 74, 109
 - colabeled, 92
 - contractible, 73, 79, 235
 - dual to cocomplex, 258
 - injectively labeled, 227
 - labeled, *see* labeled cell complex
 - locally finite, 178, 180
 - pair of, *see* cellular pair
 - pure, 96, 117
 - shellable, 269
 - weakly colabeled, 92, 97
 - weakly labeled, 79
- cellular free resolution, 63, 79, *see also* hull resolution *and* Scarf complex
 - acyclicity, 64
 - examples of, 67–71
 - minimal, 95–97, 99, 105
 - naturally occurring, 94
 - of artinian quotient, 94, 96, 105
 - of cogeneric ideal, *see* coScarf complex
 - of trivariate ideal, 99
 - of unimodular Lawrence ideal, 187
 - simple, 69, 123
 - simplicial, 69, 111, 115, 123
 - symmetric, 75
- cellular injective resolution, 232
- cellular pair, 92, 258
 - colabeled, 92
 - weakly colabeled, 92
- centrally symmetric convex body, 243
- chain complex, *see also* (co)homology
 - of cell complex, 62
 - reduced, 9, 62, 175, 233
 - relative, 235
- chain in poset
 - of Plücker coordinates, 276, 278–279, 280
 - weak order, 298, 300, 305
- character group, 192, 194
- characteristic p methods, 330
- Chow class, 172
- Chow group, 309
- chutable rectangle, 313
- chute move, 313, 314, 316, 319–322, 329
- coarse grading, 153
- coarsen, 8, 155, 263, 265, 346
- (co)boundary map, 9, 10, 63, *see also* differential
 - of cell complex, 91
- cocellular monomial matrix, 92
- cochain complex, *see also* (co)homology
 - of cell complex, 91
 - of cellular pair, 92
 - reduced, 10, 82, 83, 253
 - relative, 124
- cocomplex, *see* polyhedral cocomplex
- cocycle (i -cocycle), 11
- cogenerator, 255

- Cohen–Macaulay condition, 262–266
 and associated primes, 148
 by Gröbner degeneration, 160–161, 286
 equivalent characterizations, 263, 270
 for determinantal variety, 290
 for generic monomial quotient, 114
 for isospectral Hilbert scheme, 372
 for matrix Schubert variety, 311
 for module, 100, 269
 for monomial quotient, 103
 for normal semigroup ring, 267
 for Plücker algebra, 286
 for quiver locus, 342, 352
 for ring, 174, 263
 for Schubert determinantal ring, 328, 330, 341
 for semigroup ring, 266
 for simplicial complex, 101, 327
 for Stanley–Reisner ring, 101
 for zero-fiber of Hilb, 367, 378
 over local ring, 347
 under localization, 342, 352
 via depth, 104, 265
 via Serre’s conditions S_k , 270
 via shellability, 266, 267, 327
 (co)homological degree, 9, 66, 233, 257
 (co)homology
 Alexander duality on, 83–84
 classes of subvarieties in, 309
 commutes with direct limits, 252
 equivariant, 172, 208
 long exact sequence of, 65, 235
 of contractible space, 17, 67, 74, 236
 of links, 17, 101, 253, 267
 reduced, 9, 10, 18, 65
 relative, 92, 106, 254, 258
 sheaf, *see* sheaf cohomology
 cohull complex, 97
 coKoszul complex, 82, 83, 250
 colon ideal, 90, 91, 366, 398
 compatible fan, 199
 complete bipartite graph $K_{r,s}$, 49, 207
 complete fan, 199
 complete graph K_n , 58
 completion (of ring), 154
 complex (of modules), *see also* resolution
 acyclic, 63, 64, 93, 109, 347
 algebraic coScarf, 123
 Čech, *see* Čech complex
 cellular free, 63, 64, 73, 107, 111
 cellular injective, 218, 233
 chain, *see* chain complex
 chain map of, 19, 162, 252
 cocellular free, 93
 cochain, *see* cochain complex
 coKoszul, *see* coKoszul complex
 dualizing, *see* dualizing complex
 Eagon–Northcott, *see*
 Eagon–Northcott complex
 exact, 11
 Ishida, 257, 258, 267, 270
 Koszul, *see* Koszul complex
 minimal, 12, 109
 of flat modules, 251, 262
 of free modules, 11
 of injective modules, 216
 of localizations, 250
 Scarf, *see* Scarf complex
 stable Koszul, *see* stable Koszul
 complex
 Taylor, *see* Taylor complex
 total, 19, 252
 complexity theory, 241
 computer software, 20, 60, 75, 91, 106, 132, 148, 190, 246
 cone (over a subcomplex), 18, 29, 327
 is contractible, 17
 cone (over a variety), 196
 cone (polyhedral), 134
 dual, 200
 in fan, 199
 over polytope, 230, 238
 pointed, 134, 138–140
 rational, 134, 137–139
 simplicial, 134
 convex polyhedron, 72, 79, 144, 177, 204, 256, *see also* polytope
 convex polytope, *see* polytope
 coordinate subspace, 6, 200, 323, 324
 corner, 86, *see also* syzygy, as corner
 coScarf complex, 123, 124
 cotangent bundle, 375
 cotangent space, 362
 cover (of cell complex), 94
 Coxeter group, 309, 329, 330
 cross (crossing tile) \times , 312, 313
 cross-polytope, *see* octahedron
 cube, 81, 82, 88, 138, 198, 199, 206, 230
 cycle (i -cycle), 9, 40
 cycle notation, 291, 308

 deformation, 126, *see also* specialization
 generic, 115, 116
 of lattice module, 188–189
 of monomial ideal, 67, 115, 117, 119
 degeneracy locus, 309, 353
 degenerativity, 166, 167, 172
 degree
 of determinantal variety, 306, 308
 of monomial, 149
 of projective variety, 149
 \mathbb{Z} -graded, 165, 166, 171, 310
 degree map, 149
 fiber of, 153

- deletion (from simplicial complex), 327
- Demazure operator, 307
- depth, 104, 265
- descent, 308
- determinant, *see* minor
- determinantal ideal, 289, 295, 309, 318
 - classical, 290, 308
 - cogenerated by minor, 310
 - Cohen–Macaulayness of, 290, 325, 328, 330, 341
 - combinatorics of, 290, 312, 329
 - generated by essential minors, 294
 - Grassmannian Schubert, 172
 - ladder, 295, 309
 - of diagonal locus, 364–365, 372
 - over commutative ring, 339
 - primality of, 292, 311, 323, 330, 341
 - Schubert, *see* Schubert determinantal ideal
 - vexillary Schubert, 295, 309
- determinantal variety, *see also* matrix Schubert variety
 - classical, 290, 295, 306, 308
- diagonal locus, 356, 364, 372
- diagonal term, 278, 280
- diagram (of partial permutation), 294
 - of Zelevinsky permutation, 337–338, 348
- differential, 15, 19, 63, 233
 - horizontal, 19, 251
 - total, 19
 - vertical, 19, 251
- differential operator, 365
- dimension vector, 332
- direct limit, 252
- direct product, 219
- directed graph, 197, 353
- distraction, 360, 361, 369
- divided difference, 289, 304, 305, 309
 - isobaric, *see* isobaric divided difference
- divisible group, 192, 218
- double quiver polynomial, 346, 353
 - positive formula for, 348
 - specializes to quiver polynomial, 347
 - variables in, 343
- double Schubert polynomial, 304, 353
 - as degeneracy locus class, 309
 - as multidegree, 289, 305, 309, 324
 - double quiver polynomial from, 346
 - double Schur polynomial is, 330
 - example of, 307
 - for inverse permutation, 308
 - from quiver polynomial, 352
 - indexing of, 309
 - is universal multidegree, 308
 - is well-defined, 305
 - positive formula for, 315, 324, 329
- quiver polynomial from, 344, 347, 350
 - recursion for, 289, 304, 305
 - variables in, 305
- duality for resolutions, 91, 94–95, 106, 122, 123, 126, 228
 - in three variables, 99–100
- dualizing complex, 233, 236, 270
 - detects Cohen–Macaulayness, 266
 - for general ring, 246, 265, 270
 - Hilbert series from, 239
 - local cohomology from, 249, 254
 - Matlis dual of, 257
 - normalized, 233, 246
 - of normal semigroup ring, 234–236
- Dynkin diagram, 353
- Eagon–Northcott complex, 187
- Eagon–Reiner Theorem, 101, 106, 228
- economics, 126
- Ehrhart polynomial, 148, 229
 - as Hilbert function, 230
 - coefficients of, 230, 245
 - computing, 242, 246
 - from lattice point enumerator, 240
- Ehrhart reciprocity, 240, 246
- Ehrhart’s Theorem, 229
- eigenvector (of torus action), 192
- elbow joint (tile) \curvearrowright , 312, 313
- embedded prime, 136
- embedding dimension, 361
- equivalence of categories, 183
- equivariant Hilbert polynomial, 172
- equivariant multiplicity, 172
- essential extension, 214, 220, 222, 228
- essential set, 294, 301, 308, 309, 339
- essential submodule, 214, 221
- Euler characteristic, 66
 - \mathbb{N}^n -graded, 66, 74
- Euler’s formula, 53, 58, 120
- Ext, 198, 252, 263, 265, 268
- exterior algebra, 106
- exterior power, 274, 339
- extremal Betti number, 102, 103, 106
- extremal combinatorics, 126
- f -vector, 8, 157
- face
 - as basis vector, 63
 - dimension of, 4
 - empty, 4, 63, 66
 - flag of, 148
 - injective hull of, *see* injective hull
 - interior, 124
 - maximal, *see* facet
 - missed by polyhedron, 204, 205
 - of cell complex, 62
 - of cone, 134

- of semigroup, 133, 134
- of simplicial complex, 4, 29
- face label, 62, 217
- facet
 - of cellular pair, 92
 - of cone, 138
 - of polytope, 71
 - of simplicial complex, 5, 29
- fan, 198–199
 - nonsimplicial, 207
- Farkas’ Lemma, 134, 205, 235
- Ferrers diagram (shape), 285, 288, 305, 328, 356
- fiber product, 366, 372, 374
- field, 3
 - algebraically closed, 273, 290
 - characteristic two, 70
 - characteristic zero, 21, 352
 - finite, 6
 - of complex numbers \mathbb{C} , 191
 - positive characteristic, 277
- fine grading, 153
- flag (of faces), 73
- flag (of vector spaces), 273, 293
 - homogeneous coordinates for, 275
- flag variety, 80, 273, 275, 288, 293, 309, 330
 - degenerates to toric variety, 281
- flat degeneration, 286, 288
 - Gröbner, *see* Gröbner degeneration
 - sagbi, *see* sagbi degeneration
- flat family, 172, 360, 367
- forest, 197
- formal character, 368
- Fourier transform, 246
- free resolution, viii, 11
 - cellular, *see* cellular free resolution
 - compared to injective resolution, 211
 - equivariant, 180, 187
 - existence of finite, 156, 161
 - from staircase, 47
 - in Cohen–Macaulay criteria, 263
 - linear, *see* linear free resolution
 - minimal, 12, 14, 19, 157
 - modulo nonzerodivisor, 159, 160
 - modulo regular sequence, 346
 - of bivariate ideal, 43
 - of Borel-fixed ideal, 27–29
 - of generic ideal, *see* Scarf complex
 - of generic lattice ideal, 188
 - of generic Laurent monomial module, *see* Scarf complex
 - of residue field k , 14
 - of lattice ideal, 174, 181, 183
 - of lattice module, 183
 - of Laurent monomial module, 178
 - of quiver ideal, 346
 - of semigroup ring, 184
 - of squarefree ideal, 116
 - of trivariate ideal, 54
 - of twisted cubic, 174
 - of zero, 19, 347
 - over semigroup ring, 209
 - squarefree, 261
 - support-linear, 103, 104
 - \mathbb{Z} -graded, 30
- Frobenius power $I^{[t]}$, 18, 78, 226
- Fulton polynomial, 344, 353
- functor
 - adjoint, 216
 - Alexander duality, 106, 269
 - derived, 248
 - exact, 182, 218, 219, 269
 - faithful, 183
 - full, 183
 - fully faithful, 183
 - of points, 378
- Gelfand–Tsetlin pattern, 284, 285, 288
- Gelfand–Tsetlin toric variety, 330
- generic initial ideal (gin), 26–27, 35, 40, 45, 106
- generic matrix, 290, 332
- generic monomial ideal, 107, 109, 111–119, 122, 126, 187
 - characterization of, 76, 116–117, 126
 - Cohen–Macaulay quotient by, 114
 - free resolution of, *see* Scarf complex
 - resolution by Buchberger map, 51
 - trivariate, 50
- generic quiver representation, 333
- Geometric Invariant Theory, *see* GIT
- geometric quotient, 204
- GIT acronym, 193
- GIT quotient, 208
 - affine, 193, 194, 195, 200, 201, 203, 207
 - categorical, *see* categorical quotient
 - computing, 195
 - geometric, *see* geometric quotient
 - projective, 194, 195, 196, 204–205
- Gordan’s Lemma, 137, 148
- Gorenstein ring, 255, 266, 269, 270, 365
- Gorenstein variety, 367
- Gotzmann number, 376
- graded $_$, *see* $_$, $_*$ graded; here, $_*$ is “A-”, “arbitrarily”, “finely”, “multi”, “ \mathbb{N} _-”, “positively”, “un”, “ \mathbb{Z} -”, or “ \mathbb{Z}^d _-”, and $_$ can be as follows:
 - Betti number
 - degree
 - free resolution
 - Hilbert function
 - Hilbert series
 - homomorphism

- ideal
- injective module
- injective resolution
- \mathbb{k} -algebra
- K -polynomial module
- Nakayama's Lemma
- polynomial ring
- translate
- graded component, 150, 153
- Grassmannian, 273, 275, 288, 306
 - as Hilbert scheme, 355
 - contained in Hilbert scheme, 370
 - contains Hilbert scheme, 357–358, 368
 - degenerates to toric variety, 281
 - $G_{2,4}$, 281
 - $G_{4,8}$, 283
 - Schubert classes on, 330
- greatest common divisor, 81, 92
- green book, 19, *see* [Sta96]
- Greenlees–May duality, 106, 270
- Gröbner basis, viii, 24, 47–48, 148, 279,
 - see also* reduced Gröbner basis
 - and Hilbert scheme, 358, 360, 363, 370
 - as straightening law, 288
 - comprehensive, 25
 - for determinantal ideal, 290, 323
 - for module, 27
 - for Plücker relations I_n , 276, 277, 281
 - for quiver ideal, 339, 353
 - for syzygies of bivariate ideal, 43
 - for syzygies of Borel-fixed ideal, 30
 - for toric ideal $J_n = \text{in}_{\leq}(I_n)$, 281–283
 - geometric interpretation, *see* Gröbner degeneration
 - minimal, 282
 - short encoding for toric ideal, 244, 246
 - under weight order, 142
 - universal, for toric ideal, 244
- Gröbner degeneration, 158, 286, 311, 323
 - partial, 353
 - yields rational curve in Hilb, 360
- Grothendieck polynomial, 309
- Grothendieck–Riemann–Roch Theorem, 172
- group, *see also* orbit
 - abelian, *see* abelian group
 - algebraic, viii, 287
 - Borel, *see* Borel group
 - Coxeter, *see* Coxeter group
 - general linear GL_n , 21, 23, 208
 - representation of, 287, 288
 - symmetric, *see* symmetric group
 - torus, *see* algebraic torus
- group action, *see also* orbit
 - free, 184, 357
 - left, 299
 - of S_n and \mathbb{Z}^2 , 368
 - transitive, 301
- group algebra, 131, 161, 163, 171, 181
- h -polynomial, 8, 157, 266
- Hankel matrix, 305
- Hartshorne's counterexample, 255, 269
- Hasse diagram, 276
- Hilbert Basis Theorem, 4, 24
- Hilbert basis, 137, 138
 - as Laurent polynomial, 244
 - associated to sign pattern, 180
 - at vertex of polytope, 237
 - computing, 138–140, 141, 150, 244
 - in two dimensions, 138, 143, 146
 - of antidiagonal semigroup, 284
 - of Gelfand–Tsetlin semigroup, 285
 - parametrizes GIT quotient, 193
- Hilbert function, *see also* Hilbert series
 - multigraded, 355, 373, 375
 - positively graded, 153, 173
 - \mathbb{Z} -graded, 34, 40, 231
- Hilbert polynomial, 165, 171, 230, 231, 376
- Hilbert scheme, 21, 355
 - classical, 373, 376
 - connectedness of, 40, 360–361, 370, 376, 377
 - irreducibility of, 355, 359, 361, 363, 370, 375, 376
 - isospectral, 366, 367
 - local equations for, 357–359, 369, 375
 - most singular point of, 371
 - multigraded, 355, 373, 374, 375–376, 378
 - of points in \mathbb{C}^d , 361, 368–373
 - of points in plane, 355, 356–363, 366–367
 - of points on surface, 378
 - of points on threefold, 378
 - of subschemes of toric variety, 378
 - of \mathbb{Z} -graded ideals, 361
 - radical ideal locus, *see* radical locus
 - smoothness of, 355, 359, 361–363, 369, 375, 377
 - tangent space to, 369
 - toric, 376, 377, 378
 - universal property of, 369, 374–375
- Hilbert series, *see also* K -polynomial
 - additivity on exact sequence, 264
 - characteristic independence, 18
 - coarse, 6
 - counts torus weight spaces, 288
 - finely graded, 6, 154
 - in exact sequence, 156
 - in nonpositive grading, 149
 - modulo (non)zerodivisor, 264
 - multigraded, 153, 154–157
 - \mathbb{N}^n -graded, 6, 7, 8
 - of admissible module, 373

- of affine semigroup with units, 239
- of bivariate ideal, 42
- of canonical module, 239
- of Cohen–Macaulay module, 263
- of determinantal ideal, 330
- of graded translate, 6, 155, 157
- of ideal in semigroup ring, 228
- of indecomposable injective, 239
- of irreducible quotient, 228
- of lattice ideal, 181, 244
- of local cohomology, 247, 253–254, 269
- of modest module, 163–165, 238
- of monomial quotient, 74
- of pointed semigroup, 173
- of polynomial ring, 6, 154
- of quiver locus, 347
- of saturated semigroup, 243
- of semigroup ring, 181, 186, 230
- of Stanley–Reisner ideal, 86
- of Stanley–Reisner ring, 7, 8
- of subword complex, 330
- of tangent cone, 237
- of twisted cubic, 174
- \mathbb{Z} -graded, 6
 - of semigroup ring, 230
 - of Stanley–Reisner ring, 8
- Hilbert Syzygy Theorem, 11, 116, 156, 175, 178
- Hilbert–Burch Theorem, 174
- Hilbert–Frobenius series, 368
- Hochster’s formula, 17, 19, 85, 86, 98, 102
- Hochster, Melvin, 106
- Hom, 215, 216
- homogeneous coordinate ring (of toric variety), 71, 163, 172, 202, 208
- homogeneous polynomial, 192
- homogenization, 158–159, 162
- homological algebra, viii, ix, 269
- homological degree, *see* (co)homological degree
- homology, *see* (co)homology
- homomorphism
 - A -graded, 153
 - homogeneous, 215
 - minimal, 12
 - \mathbb{N}^n -graded, 11, 215
 - ungraded, 215
 - \mathbb{Z} -graded, 215
 - \mathbb{Z}^d -graded, 215
- Hosten–Morris number, 121
- hull complex, 79, 177
 - characteristic independence, 75
 - computing, 181
 - contains Scarf complex, 111–112
 - of artinian ideal, 76–78
 - of generic ideal, 111, 117
 - of lattice module, 180–181, 185
 - of Laurent monomial module, 178
 - of monomial ideal, 73
- hull resolution, 71–78, 109
 - not every cellular resolution is, 98
 - of lattice module, 184
 - of Laurent monomial module, 178, 188
 - of monomial ideal, 73
 - of semigroup ring, 184
- hyperplane arrangement, 79, 80
- hypersimplex, 180
- ideal, *see also* monomial ideal
 - admissible, *see* admissible ideal
 - antidiagonal, 318, 319, 321–323, 330
 - binomial, *see* lattice ideal
 - Borel-fixed, *see* Borel-fixed ideal
 - determinantal, *see* determinantal ideal
 - face, 19, 248, 255, 269
 - finitely generated, 4
 - G -stable, 193
 - GL_n -fixed, 23, 33
 - in semigroup, 133
 - initial, *see* initial ideal
 - irreducible, 87, 91, 211, 225
 - irrelevant, *see* irrelevant ideal
 - Jacobian, 363
 - lattice, *see* lattice ideal
 - lex-segment, 34–39
 - maximal, *see* maximal ideal
 - monomial, *see* monomial ideal
 - multigraded, 193
 - \mathbb{N}^n -graded, 4
 - permutohedron, 68, 69, 75, 97, 99, 123
 - prime, 135, 195, 288
 - principal, 136, 147, 148, 174, 209, 228, 269
 - pure, 171
 - radical, *see* radical ideal
 - squarefree, *see* squarefree ideal
 - stable, 28, 40
 - Stanley–Reisner, *see* Stanley–Reisner ideal
 - torus-fixed, *see* monomial ideal
 - tree, 68, 69, 80, 97, 99, 109, 123
- inclusion–exclusion, 42–43, 67, 74, 210
- indecomposable injective module, 103, 212, 213–214, 247, 256
 - arbitrarily graded, 228
 - homomorphism of, 216
 - is homologically injective, 219
 - supported on I_Δ , 248
- independent paths, 57
- initial algebra, 279, 281
 - of Plücker algebra, 280, 281, 286
- initial complex, 142, 144–146
- initial form, 142
- initial ideal, 24
 - for Plücker relations, 277–279, 282
 - of determinantal ideal, 311, 323

- of distraction, 360
- of lattice ideal, 142–146, 148
- of toric ideal, 148
- squarefree, 187, 307, 323
- under weight order, 142
- initial module, 27, 30, 166
 - as special fiber, 158
 - under weight order, 158, 159–161
- initial term, 24, 279
 - of Plücker product, 278–279, 280
- injective hull
 - arbitrarily graded, 228
 - as essential extension, 214
 - of face, 212, 213, 216, 221, 233, 239
 - of module, 221, 222
 - of residue field k , 104, 265
- injective module, 211, 213, 214–223
 - arbitrarily graded, 227
 - characterizations of, 220
 - decomposition as direct sum, 223
 - finely graded, 228
 - homomorphism of, 216, 217
 - indecomposable, *see* indecomposable injective module
 - product of, 219
 - ungraded, 251, 258
- injective resolution, 103–104, 222
 - arbitrarily graded, 227
 - compared to free resolution, 211
 - computing, 270
 - existence and uniqueness, 222
 - finite, 265
 - from cell complex, 227
 - homological invariants from, 222
 - in Alexander duality, 95
 - in Cohen–Macaulay criterion, 263
 - infinite, 265
 - local cohomology from, 248
 - minimal, 222, 223, 224, 265
 - of canonical module, 233, 235, 266
 - of generic monomial ideal, 228
 - ungraded, 252
 - \mathbb{Z}^d -graded, 247
 - \mathbb{Z}^n -graded, 126
- inner normal vector, 77, 78, 120, 139, 197
- integer programming, 143, 148, 190
- integral domain, 131
- interior lattice point enumerator, 236
- intron, 316, 317
- intron mutation, 316, 317, 329
- invariant theory, 191, 378
- irreducible component, 87, 88
 - as Bass number, 223, 224
 - as facet, 58, 96, 117, 226
 - as outer corner, 47, 100, 226
 - as upper bound for syzygy degree, 117
 - as vertex of cell complex, 124
 - of artinian ideal, 104
 - of generic ideal, 114
 - under specialization, 125
 - upper bound on number of, 120
- irreducible decomposition, 87, 106
 - computing, 87, 91, 106, 212, 228
 - from cellular resolution, 96–97, 125
 - from staircase, 47
 - in semigroup ring, 211, 212
- irredundant, 87, 105
 - of bivariate ideal, 43
 - of Borel-fixed ideal, 24
 - of generic ideal, 114, 117
 - of non-monomial ideal, 91, 228
 - uniqueness, 90, 211
- irreducible hull, 222
- irreducible monomial ideal
 - from indecomposable injective, 214
 - heuristic illustration of, 215
 - in polynomial ring, 43, 87
 - in semigroup ring, 137, 210, 225
 - testing for, 212
- irreducible quotient, 210, 222
- irreducible representation, 288
- irreducible resolution, 210, 228, 270
 - computing, 228
 - existence and uniqueness, 211, 223, 224
 - over polynomial ring, 225
- irrelevant complex, 4, 121
 - as a link, 17
 - (co)homology of, 9, 10, 14, 83
 - Euler characteristic of, 66
- irrelevant ideal
 - of compatible fan, 199, 202, 205, 208
 - of \mathbb{N} -graded ring, 195, 206
 - of Plücker algebra, 287
 - of quotient ring, 202
 - of toric variety, 71, 260
- isobaric divided difference, 307
- Jacobi–Trudi formula, 305
- join \vee , 92, 282
- k -algebra
 - \mathbb{N}^n -graded, 4
 - \mathbb{Z} -graded, 266
 - \mathbb{Z}^d -graded, 215
- K -polynomial, 172, *see also* Hilbert series
 - additivity on exact sequence, 169, 306
 - as Euler characteristic, 67, 74, 113
 - at vertex of polytope, 237
 - calculation of, 106, 158
 - from staircase, 47
 - multigraded, 161
 - \mathbb{N}^n -graded, 7
 - of bivariate ideal, 43
 - of Borel-fixed ideal, 28

- of free module, 161
- of Frobenius power $I^{[t]}$, 18, 78
- of generic ideal, 113
- of matrix Schubert variety, 307, 309
- of modest module, 164–165
- of prime monomial quotient, 168
- of quiver locus, 347
- of residue field k , 158, 161
- of semigroup ring, 173, 186
- of Stanley–Reisner ring, 7, 86
- of subword complex, 106
- of trivariate ideal, 58
- positively graded, 157
- records Betti numbers, 157
- under change of grading, 171
- under Gröbner degeneration, 162
- universal, 172
- K -theory, 148, 157
- equivariant, 106, 172, 246
- Koszul complex, 13, 30, 264, *see also*
 - coKoszul complex
 - as hull complex, 74, 76
 - as linear free complex, 31
 - as minimal free resolution, 14, 158, 168, 175
 - compared to coKoszul complex, 82, 83
 - Matlis dual of, 218
- Koszul simplicial complex
 - lower: $K_{\mathbf{b}}(I)$, 84
 - upper: $K^{\mathbf{b}}(I)$, 16, 18, 31, 52, 84, 117
- Krull dimension, 142, 231, 254, 262, 301
- labeled cell complex, 62, 92, *see also*
 - simplicial complex, labeled
- lace, 332, 352
- lace array, 333, 334–335, 337, 338
 - from permutation, 344
- lacing diagram, 331, 332, 333–336
 - as component of degeneration, 353
 - double Schubert product from, 350
 - from pipe dream, 349–350, 353
 - minimal, 350, 352
- Lagrange interpolation, 242
- Laplace expansion, 293
- lattice (combinatorial)
 - Boolean, 81
 - distributive, 92, 281–282, 288
- lattice (in \mathbb{Z}^n), 130
 - generic, 188, 189, 190
 - unimodular, 187
- lattice ideal, 129, 130, 139, 176
 - as point on toric Hilbert scheme, 377
 - computation of, 132, 148, 244
 - from lattice module, 183
 - from meet-join lattice, 288
 - generic, 188, 189
 - in Laurent polynomial ring, 192
 - initial ideal of, *see* initial ideal, of lattice ideal
 - is $\langle x_1 \cdots x_n \rangle$ -saturated, 132
 - prime, 131
 - toric ideal is, *see* toric ideal
- lattice module, 179, 180–187
 - generic, 188
 - presentation of, 179
- lattice point enumerator, 236, 237–243
- Laurent monomial module, 176, 177–180, 190
 - generic, 187
- Laurent polynomial, 157, 163
- Laurent polynomial ring, 131, 176, 179
 - as coordinate ring of torus, 192
- Laurent series, 153, 154–155, 163
 - summable, 163, 238, 239
 - supported on translates, 155, 156
- Lawrence ideal, 139, 147, 148
 - unimodular, 187, 190
- Lawrence lifting, 187
- lcm-lattice, 74, 79, 80
- leading term, *see* initial term
- least common multiple, 18, 42, 61, 74, 81, 92, 107, 109, 110, *see also* lcm-lattice
- length
 - of free resolution, 11
 - of lacing diagram, 350
 - of module, 165, 356
 - of partial permutation, 289, 294, 325
- Lenstra–Lenstra–Lovasz algorithm, 243
- L’Hôpital’s rule, 240, 241
- lineality, 134, 139, 233, 239, 254, 257
- linear extension, 277
- linear free resolution, 30, 70, 101, 105
- linear programming, 143, 145, 148
- link, 17, 86, 101, 105, 181, 253, 327
- Littlewood–Richardson rule, 288
- local cohomology, 247, 248, 249–262, 269
 - computing, 256, 262
 - finite data structure for, 255–256
 - grading on, 251
 - in Cohen–Macaulay criteria, 263, 265, 267
 - module structure, 269
 - not finitely cogenerated, 250, 255
 - not finitely generated, 250
 - of canonical module, 249, 254–255, 269
 - of semigroup ring, 255, 258
 - of Stanley–Reisner ring, 253
 - via Čech cohomology, 251, 265
 - via Ext, 252
 - via generalized Čech cohomology, 260
 - via Ishida complex, 258, 267
 - with maximal support, 258
 - with Stanley–Reisner support, 254, 260
- local duality, 106, 233, 265, 270

- with monomial support, 270
 - localization
 - along a face, 222
 - at maximal ideal, 346
 - exactness of, 347
 - flatness of, 218
 - monomial, 105
 - of \mathbb{N} -graded ring, 196
 - of polynomial ring, 342, 352
 - of semigroup, 134, 258
 - of semigroup ring, 216, 257, 258
 - of Stanley–Reisner ring, 253
 - long word (permutation), 291, 304
 - lower triangular matrices, *see* Borel group
 - Macaulay’s Theorem, 34
 - Macdonald polynomial, 368, 378
 - manifold, 86
 - mapping cocylinder, 162
 - matching (of a graph), 336
 - Matlis duality, 216, 217, 257, 269
 - as Hom into injective hull of \mathbb{k} , 219
 - in Alexander duality, 226
 - in Cohen–Macaulay criterion, 263
 - matrix Schubert variety, 289, 290, 309
 - associated to Grassmannian, 306
 - boundary components of, 302–304, 306
 - containment among, 295
 - dimension of, 295, 298, 301
 - double Schubert polynomial from, 305
 - equations defining, 291–295
 - fixed by adjacent transposition, 302
 - for long word w_0 , 291
 - for Zelevinsky permutation, 342
 - from quiver locus, 338, 341
 - Gröbner degeneration of, 323, 353
 - is Borel orbit closure, 299–302
 - is Cohen–Macaulay, 311
 - is irreducible, 295, 301
 - nonzero function Δ on, 303, 306
 - partial permutations in, 296
 - Schubert polynomial from, 307
 - Schubert variety from, 293
 - maximal ideal
 - in affine semigroup ring, 147, 257
 - in polynomial ring, 87, 263, 369
 - localization at, 346
 - of artinian ring, 356
 - of identity in $B \times B_+$, 303
 - of partial permutation, 303
 - of partition on Hilb (\mathfrak{m}_λ), 361
 - of smooth point, 301
 - meet \wedge , 92, 281
 - Menger’s Theorem, 57
 - minimal generator
 - as facet of cell complex, 124
 - as inner corner, 45, 57
 - as vertex of hull complex, 73
 - as vertex of polyhedron, 74
 - computing for toric ideal, 244
 - number of, 14, 38
 - of cogeneric ideal, 123
 - of diagonal locus ideal, 364, 373
 - of graded component $S_{\mathbf{a}}$, 150
 - of irreducible ideal, 212
 - of lattice ideal, 175, 181
 - of lattice module, 179
 - of Laurent monomial module, 177, 178
 - of maximal ideal, 147, 361
 - of module, 156, 221
 - of monomial ideal, 4, 28, 42, 72, 88, 358
 - of pointed semigroup, 137, 173
 - of $S_{\mathbf{0}}$ -algebra $S_{(\mathbf{a})}$, 195
 - of semigroup ring, 147
 - of squarefree ideal, 81
 - upper bound on number of, 125
- Minkowski sum, 245
- Minkowski’s Theorem, 243
- minor (of a graph), 58
- minor (of a matrix), 274, 291
 - 2×2 , 187, 206
 - defined by rank condition, 318
 - generic, 275
 - in Gröbner basis, 323, 324, 339, 353
 - in product of generic matrices, 333
 - in product of matrices, 331
 - in product of two matrices, 335–336
 - in sagbi basis, 280, 324
 - maximal, 176, 274, 290, 308
- minor (miracle), 180
- mitosis, 314, 315, 317, 329, 330
- Möbius function, 232, 243
- modest module, *see* module, modest
- module
 - Cohen–Macaulay, 100, 263, 269
 - equivariant, 182–183
 - filtration by prime quotients, 169
 - finite-length, 165
 - finitely generated, 12, 14, 150, 151, 153, 161, 165
 - flat, 218, 219, 264, 373
 - free, 11, 27, 156, 263
 - graded, 153
 - homologically injective, 218, 219–221
 - infinitely generated, 152, 153, 177
 - injective, *see* injective module
 - is submodule of injective, 220
 - locally free, 373, 374
 - modest, 163, 164–165, 238
 - multigraded, 153
 - \mathbb{N}^n -graded, 6
 - of Laurent series, 163

- positively graded, 153, 156–158, 160–161, 373
- Q -graded, 211, 222, 224
- Q -graded part of, 214
- ungraded, 251, 252, 258
- ungraded free, 12
- \mathbb{Z} -graded, 263
- \mathbb{Z}^n -graded, 182
- \mathbb{Z}^n/L -graded, 182
- moment curve, 119
- moment polytope, 71
- monomial, 3, 149
 - in semigroup ring, 133
 - squarefree, 4
 - standard, *see* standard monomial
- monomial ideal
 - artinian, 47, 50, 54, 76, 77–78, 104
 - as initial ideal, 144
 - as point on Hilb, 356, 360, 363, 367
 - Borel-fixed, *see* Borel-fixed ideal
 - cogeneric, 107, 122
 - Cohen–Macaulay quotient by, 103
 - generic, *see* generic monomial ideal
 - graded translate of, 177
 - in polynomial ring, 3
 - in semigroup ring, 133, 135, 209
 - infinite periodic, 176
 - irreducible, *see* irreducible monomial ideal
 - is fixed by torus, 22, 363
 - maximal, 87, 147, 257
 - most singular on Hilb, 371
 - neighborly, 121
 - prime, 5, 81, 134, 166
 - resolves itself, 61
 - rigid, 59
 - squarefree, *see* squarefree ideal
 - stable, *see* ideal, stable
 - strongly generic, 50, 60, 109, 126, *see also* generic monomial ideal
 - trivariate, 44–47, 49–59, 67, 285
 - with given Hilbert function, 377
- monomial label, 62, 217
- monomial matrix, 20, 174
 - cellular, 62
 - cellular injective, 217, 233
 - differential without, 13, 63, 64, 107
 - for free modules, 12, 215, 227
 - for injective modules, 215, 217, 227
 - for \mathbb{Z}^n -graded localizations, 250, 259
 - minimal, 12
- monomial order, *see* term order
- morphism
 - fibers of, 204
 - G -equivariant, 203
 - of schemes, 374
 - of varieties, 201, 351, 372
 - projective, 195
- mountain topography, 52
- multidegree, 149, 167, 172, 286
 - additivity, *see* additivity
 - degenerativity, *see* degenerativity
 - existence and uniqueness, 166
 - is polynomial (not integer), 304, 310
 - of codimension r module, 169
 - of graded translate, 169
 - of matrix Schubert variety, 289, 304–307
 - of prime monomial quotient, 168
 - of quiver locus, 343
 - of twisted cubic, 169–170, 171
 - of variety, 167
 - positivity, 171, 311
 - under change of grading, 171
 - universal, 172
- multigrading, 149, 172, 304, 375
- multiple Proj, 288
- multiplicity (of M at \mathfrak{p}), 165
- $n!$ Theorem, 266, 355, 363, 365, 367, 368
- $(n+1)^{n-1}$ Theorem, 266, 363, 365, 367
- n -connected, 59
- \mathbb{N} -grading (arbitrary), 263–265
- Nakayama’s Lemma, 162
 - for semigroup rings, 147
 - nonstandard version, 264
 - positively graded, 155
 - \mathbb{Z} -graded, 155
- near-cone, 29
- nerve (of a cover), 94, 95, 176
- Newton polytope, 71
- nilpotent element, 356
- nonface, 5, 17
- nonzerodivisor, 262, 264
- normal fan, 145, 146, 199, 205
- normalization, 140, 141, 147, 230, 231
- octahedron, 18, 66, 68, 71, 82, 146, 148, 199, 230, 245, 246
- offspring, *see* pipe dream, offspring of one-line notation, 291
- open cover, 196, 208, 358
- opposite big cell, 345
- opposite Schubert cell, 341, 345
- optical illusion, 81, 88, 90
- optimal vector, 142
- orbit
 - of algebraic group, viii, 301
 - of Borel group, 289, 300–301
 - of general linear group, 208, 353
 - of subgroup of torus, 193, 194, 200, 203, 204, 207, 363
- order complex, 126, 279
- order dimension, 60

- order ideal, 64, 368
- orientation, 62
- oriented matroid, 72
- outer normal vector, 76, 205
- parabolic subgroup, 39, 341, 342
- part (of a partition), 285
- partial flag variety, 341, 345
- partial permutation, 289, 290, 291
 - drawing of, 312
 - extension of, 292, 293, 301, 304, 312
 - family connecting pair of, 300
 - in lacing diagram, 331–332, 352
 - indexes Borel orbit, 300
 - length after switching rows, 296–297
- partition, 285, 288, 305, 328, 356, 361, 364, 365
- permutation, *see also* partial permutation
 - conventions for, 291, 325
 - Grassmannian, 308, 328
 - vexillary, 295, 309
 - Zelevinsky, *see* Zelevinsky permutation
- permutohedron, 68, 80, 123
- picture space, 330
- pipe dream, 312, 313–324, *see also*
 - reduced pipe dream
 - barren, 314
 - coordinate subspace from, 320
 - offspring of, 314
 - subword from, 325–326
 - top-justified, 320, 321
- pipe dream formula, 353
- planar graph, 60
- planar map, 51
 - axial, 99
 - dual of, 99
 - labeled, 51, 52, 54
 - radial, 99
- plane partition, 369
- Plücker algebra, 275
 - is Cohen–Macaulay, 286
 - presentation of, 276
 - spector of, 287
- Plücker coordinates, 273, 275
 - as generic minors, 275
 - form sagbi basis, 280, 324
 - relations among, 277
 - represent flags, 275, 293
 - represent subspaces, 274, 357
- Plücker relations, 276, 277–279, 373
- polarization, 44–45, 59–60, 116
- polyhedral cell complex, *see* cell complex
- polyhedral cocomplex, 258
- polyhedral subcomplex
 - in a cover, 94
 - of cone, 248, 254
 - of polytope, 73, 234, 235
- polyhedral subdivision, *see* subdivision
- polyhedron, *see* convex polyhedron
- polynomial ring, 3
 - bivariate, 42
 - is semigroup ring, 129
 - multigraded, 149, 191
 - \mathbb{N}^n -graded, 4
 - positively graded, 151, 152, 153, 173, 195, 262, 263
 - trivariate, 41
 - with real exponents, 52, 115
 - \mathbb{Z} -graded, 21, 191, 230
 - \mathbb{Z}^n/L -graded, 182
- polynomial time, 241–245
- polytope, 61, 62, 77
 - 3-dimensional, 62
 - boundary of, 145
 - cyclic, 114, 119, 120–121
 - interior lattice points in, 236–238, 240
 - lattice, 148, 197, 199, 229–232, 237, 240–242, 246
 - lattice points in, 153, 229, 236–243
 - neighborly, 120, 121
 - normal, 232, 245
 - polar, 145, 258
 - rational, 245, 246
 - section of cone, 233, 254
 - simple, 70, 71, 75, 82, 122, 145
 - simplex, 231
 - simplicial, 82, 102, 122
 - transportation, 207
 - triangulating, 231
 - volume of, 230, 246
- poset
 - Bruhat, *see* Bruhat order
 - face, 72, 178, 232, 243, 258
 - of injective submodules, 221
 - of least common multiples, *see*
 - lcm-lattice
 - \mathcal{P} of Plücker coordinates, 276–277, 278–279, 281, 286
 - pointed semigroup as, 137
 - product of intervals, 90
 - weak order, *see* weak order
- positive multigrading, 151, 194, 198, 375
- positively graded, *see* graded
- power series
 - modest, 164
 - supported on semigroup, 154
- power sum, 364, 365, 368
- precedes (\preceq), 64
- presentation (of group), 192
- primary decomposition, 24, 146
 - characteristic dependent, 131
 - in semigroup ring, 133, 135–137
- prime avoidance, 263
- primitive integer vector, 205

- primitive lattice vector, 180, 181
- Proj, 195, 256, 281
- projective dimension, 100, 160–161
 - dual to regularity, 102–104
 - of generic ideal, 114
 - of lattice ideal, 175–176
- projective space \mathbb{P}^r , 6, 198, 200, 206, 275, 287
 - homogeneous coordinates, 273, 274
- projective spectrum, *see* Proj
- proper extension, 214
- Q -set, 147
- quasi-polynomial, 245
- quiver, 353
- quiver ideal, 331, 333, 352
 - determined by lace array, 335
 - multigrading for, 343
 - primality, 336, 341–343, 352
 - quiver polynomial from, 343
 - to Schubert determinantal ideal, 339
- quiver locus, 331, 333, 352
 - Cohen–Macaulayness, 341–343, 352
 - for other types of quiver, 353
 - matrix Schubert variety from, 338, 341
- quiver polynomial, 288, 331, 343, 353
 - double, *see* double quiver polynomial
 - K -theoretic, 353
 - positive formula for, 347, 349, 350, 353
- quiver representation, 208, 332–333, 334–335, 349
 - finite type, 352
 - indecomposable, 333, 334, 352
- quiver variety, 208, *see also* quiver locus
 - and toric variety, from quiver
- quotient semigroup, 134
- radical component, 371, 372
- radical ideal, 171
 - corresponds to algebraic set, 6
 - failure to be, 369
 - from squarefree initial ideal, 307, 323
 - of n points, 356, 361, 369
 - support on, 248
- radical locus, 357, 360, 362, 369, 370, 371
- rank array (for partial permutation), 290, 296–298
 - of Zelevinsky permutation, 340
- rank array (for quiver), 333, 334–335
 - from permutation, 344, 352
 - irreducible, 335
 - minimal lacing diagram for, 350, 352
 - prime quiver ideal from, 342
 - quiver polynomial from, 343
 - Zelevinsky permutation from, 337–338
- rank-nullity theorem, 66, 156
- ratio formula, 353
- rational curve, 359, 360
- rational function, 163
 - as generating function, 173, 229
 - equated with Laurent polynomial, 237
 - equated with Laurent series, 154
 - in power series ring, 165, 167
 - short, 229, 243, 244, 246
- rc-graph, 329
- rectangle array, 333, 334, 337, 352
- reduced homology or cohomology, *see* (co)homology, reduced
- reduced expression, 305, 311, 325, 329
 - in Coxeter group, 330
 - reverse triangular, 326
- reduced Gröbner basis, 25
 - computing for toric ideal, 244
 - for module, 27, 156
 - for Plücker relations I_n , 282, 283
 - for syzygies of Borel-fixed ideal, 31
 - for toric ideal $J_n = \text{in}_{\leq}(I_n)$, 282
 - is homogeneous, 172
 - under weight order, 142
 - uniqueness of, 39, 148
- reduced pipe dream, 312, 326, 329
 - as coordinate subspace, 318
 - as facet complement, 318, 322, 323
 - as Gelfand–Tsetlin face, 288, 330
 - as monomial ideal generator, 318
 - as prime component, 311, 318
 - as Young tableau, 328
 - bottom, 328
 - crossing tiles \times in, 328
 - double Schubert monomial from, 324
 - for long word w_0 , 312, 317, 326
 - for Zelevinsky permutation, 348–350
 - generating all, 314–315
 - involution on, 315, 317
 - offspring of, 317
 - quiver monomial from, 348, 349
 - reduced subword from, 326
 - Schubert monomial from, 315, 317
 - top, 321, 328
- Rees algebra, 372–373
- regular function, 341, 342, 367
- regular sequence, 262
 - criterion for being a, 347
 - depolarization by, 44, 60
 - in Cohen–Macaulay criteria, 263–265, 342
 - quotient by preserves acyclicity, 346
- regular subdivision, 78, *see also* triangulation, regular
- regularity, 45, 101, 102–104
 - multigraded, 378
- Reisner’s criterion, 101, 102, 106, 266, 267
- Reisner (reess’-nør), Gerald, 106

- representation theory, 191, 284, 287, 298, 353
- representation
 - of abelian group, 192
 - of quiver, *see* quiver representation
 - of symmetric group, 368
- resolution, *see also* complex (of modules)
 - Alexander duality for, *see* duality for resolutions
 - by planar map, 51, 52, 54, 67, 99
 - cellular $_$, *see* cellular $_$ resolution
 - cocellular, 93, 94–98
 - cohull, 97, 98, 105, 123
 - coScarf, 123–125, 126
 - Eliahou–Kervaire, 33, 40
 - free, *see* free resolution
 - hull, *see* hull resolution
 - injective, *see* injective resolution
 - irreducible, *see* irreducible resolution
 - Scarf, *see* Scarf complex
 - Taylor, *see* Taylor resolution
 - weakly cellular, 79, 98, 105
 - weakly cocellular, 93, 94, 96, 97
- resolution of singularities, 196
- restriction (of simplicial complex), 85, 86
- reverse square word, 326, 327
- ridge, 233, 329
- rigid embedding, 60
- ring
 - arbitrary (commutative), 84, 129, 208, 215, 216, 227, 250, 251, 252, 304, 309, 339, 352, 373
 - Cohen–Macaulay, 263
 - cohomology, 208, 288
 - determinantal, *see* ring, Schubert determinantal
 - face, 19, 248, 269
 - group, *see* group algebra
 - K -, 208, 309
 - Laurent polynomial, *see* Laurent polynomial ring
 - local, 302, 303, 347, 356, 362
 - of global sections, 367
 - of invariants, 193, 364
 - polynomial, *see* polynomial ring
 - power series, 6, 154
 - regular, 209
 - regular local, 301, 304
 - Schubert determinantal, 286, 325, 328
 - semigroup, *see* semigroup ring
 - Stanley–Reisner, *see* Stanley–Reisner ring
- Robinson–Schensted–Knuth
 - correspondence, 329
- rook placement, 291
- root of unity, 194
- s -pair, 47, 48, 121
- sagbi acronym, 279, 288
- sagbi basis, 273, 279, 281, 288
 - for Plücker algebra, 280
- sagbi degeneration, 281, 286
- sans serif font, 125
- saturation of a semigroup, *see* semigroup, saturation of
- saturation of an ideal, 132, 207, 366, 398
- scalar entries, 12
- Scarf complex, 107, 110, 111–114
 - algebraic \mathcal{F}_{Δ_I} , 111
 - can be disconnected, 110
 - characteristic independence of, 112
 - is contained in hull complex, 111
 - of deformation, 115–117
 - of generic artinian ideal, 113, 228
 - of lattice, 190
 - of lattice ideal, 188
 - of Laurent monomial module, 188
 - of monomial ideal, 187
 - purity of, 113
- Scarf triangulation, 227
- scheme, 202, 207, 352, 356, 366, 367, 369, 374, 376
 - projective, 375
- Schlegel diagram, 73, 77, 114
- Schreyer’s algorithm, 32, 156
- Schubert determinantal ideal, 290, 292, 293–295, *see also* determinantal ideal
 - for Zelevinsky permutation, 339
 - from quiver ideal, 336, 339
 - primality of, 323
 - universal multigrading for, 308
- Schubert determinantal ring, *see* ring, Schubert determinantal
- Schubert polynomial, 304, 309, *see also* double Schubert polynomial
 - and Gelfand–Tsetlin patterns, 288
 - as multidegree, 307, 323
 - coefficients of, 311
 - combinatorics of, 286, 312
 - positive formula for, 315, 323, 329
 - quantum, 353
 - recursion for, 304, 305, 311
 - Schur polynomial is, 305, 328
 - stable, 330
 - universal, 353
- Schubert variety, 288, 289, 293, 309, 330
 - in partial flag variety, 341, 345
- Schur function, 172, 368
- Schur polynomial, 290, 305, 306, 328, 330, 353
- sector, 249, 255, 256
- sector partition, 255, 256, 270
- Segre variety, 206
- semi-invariant, 353

- semigroup, *129*, *see also* semigroup ring
 - affine, *129*, *131*, *133–135*, *137–141*, *148*, *173*, *176*, *187*, *209*, *233*, *247*
 - antidiagonal, *284*, *286*
 - can be a group, *133*
 - cancellative, *129*
 - characteristic function of, *376*, *377*
 - cone over polytope, *230*, *232*
 - embedding in $\mathbb{N}^{\#\text{facets}}$, *139*
 - embedding in \mathbb{N}^{rank} , *140*, *152*
 - finitely generated, *129*
 - Gelfand–Tsetlin, *284–286*
 - holes in, *148*
 - nonsimplicial, *270*
 - pointed, *133*, *134*, *140*, *141*, *148*, *152*, *154*, *171*, *173*, *178*, *254*, *257*
 - saturated, *137*, *140*, *141*, *147*, *150*, *233–236*, *254*, *269*, *286*
 - saturation of, *140*, *141*, *147*, *230*, *268*
 - $\sigma^\vee \cap L$ for cone σ , *200*, *204*, *205*
 - unsaturated, *270*
 - vertex, *see* vertex semigroup
- semigroup ring, *129*, *see also* lattice ideal
 - (anti)diagonal, *284*
 - affine, *133–137*, *140–141*, *236*, *248–250*, *256–259*, *266*
 - $\mathbb{C}[\sigma^\vee \cap L]$ for cone σ , *201*, *203–205*
 - Cohen–Macaulay, *266–267*
 - completion of, *154*
 - dimension of, *131*
 - from lattice module, *181*
 - Gelfand–Tsetlin, *286*
 - integral domain, *131*
 - intersecting ideals in, *135–136*
 - normal, *140*, *150*, *193*, *232*, *254–256*, *267*, *270*, *286*
 - normalization of, *see* normalization over the integers \mathbb{Z} , *129*, *154*
 - presentation of, *130–131*
 - vertex, *237*
- semistandard monomial, *279*, *283*
- semistandard tableau, *see* tableau
- Serre’s condition S_k , *148*, *270*
- shadow, *200*
- shape, *see* Ferrers diagram
- sheaf, *71*, *172*, *208*, *367*, *374*
- sheaf cohomology, *71*, *256*, *269*, *330*
- shear, *144*, *350*
- shelling, *267*
- shuffle, *277*
- sign convention, *9*
- simple reflection, *325*
- simplex
 - in simplicial complex, *see* face
 - polytope, *see* polytope, simplex
- simplicial complex, *4*, *9–11*
 - antidiagonal, *see* antidiagonal complex
 - as polyhedral cell complex, *62*
 - associated to affine semigroup, *175*
 - bijection with squarefree ideals, *5*
 - Cohen–Macaulay, *101*, *266*, *327*
 - dimension of, *4*
 - irrelevant, *see* irrelevant complex
 - labeled, *13*, *107*, *see also* labeled cell complex
 - of faces missed by polyhedron, *205*
 - of poset chains, *see* order complex
 - pure, *45*, *267*, *323*, *327*, *329*
 - shellable, *266*, *267*, *270*, *327*, *329*, *330*
 - shifted, *29*, *31*
 - Stanley–Reisner, *142*, *199*, *253*, *318*
 - subword, *see* subword complex
 - vertex-decomposable, *327*, *329*, *330*
 - void, *see* void complex
 - without pair of covering faces, *121*
- simplicial fan, *199*, *204*
- slope variety, *330*
- Smith normal form, *131*, *133*, *148*
- smooth fan, *199*
- smooth point, *301*, *363*
- socle, *104*, *265*
 - of local cohomology, *255*, *256*, *270*
- source degree, *12*
- Spec, *193*, *281*
- special fiber, *158*, *353*, *367*
- specialization, *52*, *see also* deformation
- spectator (SpecTor), *202*, *208*, *256*, *281*
 - of Plücker algebra, *287*, *288*
- spectral sequence, *20*
- spectrum, *see* Spec
- sphere, *83*, *84*, *86*, *102*, *145*, *269*, *329*, *330*
- spherical variety, *288*
- squarefree ideal, *4*, *5–8*, *16–19*
 - advantage of, *41*
 - as polarization, *44*
 - associated to polytope, *70*, *75*, *82*, *259*
 - encodes fan, *199*
 - from determinantal ideal, *318*
 - from lattice ideal, *146*
 - generated in degree d , *71*
 - in Plücker algebra, *287*
- squarefree module, *106*
- squarefree vector, *5*, *75*
- stable Koszul complex, *250*
- staircase diagram, *42*, *45*, *69*, *74*, *88*, *89*, *98*, *99*, *113*, *126*, *177*, *179*, *185*, *226*, *261*, *356*, *362*, *369*
- staircase surface, *49*, *50*, *59*, *60*, *68*, *99*, *100*, *105*
- standard monomial, *158*, *211*, *215*, *285*
 - as basis element, *158*, *357*
- standard monomial theory, *288*
- standard \mathbb{Z} -grading, vii, *21*, *171*, *230*, *290*, *304*, *361*

- Stanley–Reisner ideal, 3, 5, 6, 18, 19
 - in Alexander duality, 16, 82
 - of initial complex, 142
 - of order complex, 279
 - of real projective plane, 69, 80
- Stanley–Reisner ring, 3, 5, 19
 - Cohen–Macaulay, 101, 267
 - Gorenstein, 269
 - of real projective plane, 70, 75
- star, 105
- Steinitz Theorem, 62
- stick twisted cubic, 82, 102
- straightening law, 288, 329
- strand (= lace), 352
- strictly divides, 109, 117
- strong deformation, 52, 54, 60, *see also* deformation
- subalgebra basis, *see* sagbi basis
- subdivision, *see also* triangulation
 - barycentric, 69, 73, 94, 110
 - infinite periodic, 180
 - of polyhedral cell complex, 77
 - of polytope, 77
 - of simplex, 77, 117
 - of torus, 188
 - regular, *see* regular subdivision
- subword, 326–327
- subword complex, 326, 327–328, 330
- summable, *see* Laurent series, summable
- support Γ_I on ideal, 248, 251, 252
 - on maximal ideal, 257, 265
- support of a vector, 7
 - full, 77, 105
- support-regularity, 103
- suspension (of a graph), 53
- sweeping, 299
- symmetric function, 305, 353, 355, 368
 - complete homogeneous, 308
 - elementary, 308
 - Stanley, 330
- symmetric group, 291, 325, 337, 356, 364, 368
- symmetric product, 356, 364, 372
- symplectic geometry, 284, 288
- system of parameters, 262, 263–265, 269
- syzygy, *see also* Betti number
 - as corner, 43, 49, 52, 54, 56, 57
 - as face of cell complex, 99, 119
 - from Buchberger edge, 48
 - from planar map, 47
 - in linear free resolution, 30
 - of bivariate ideal, 43
 - of Borel-fixed ideal, 30
 - of lattice ideal, 174
 - of lattice module, 179
 - of Laurent monomial module, 178, 187
 - of lex-segment ideal, 35
 - of monomial ideal, 187
 - of trivariate ideal, 53
 - of twisted cubic, 174
 - of unimodular Lawrence ideal, 187
- syzygy module, 11, 14, 48
 - from Buchberger graph, 48
- tableau, 276, 280, 281, 282, 288, 306, 328
- tangent cone, 233, 234, 235, 237, 238
- tangent space, 369, 371
- target degree, 12
- Taylor complex \mathcal{F}_Δ , 107, 108–110, 111, 115
- Taylor resolution, 67, 74, 80, 108, 111
- tensor product, 15, 153, 155, 182, 216
- term order, *see also* weight order
 - antidiagonal, 280, 323
 - diagonal, 280, 282, 324
 - for free module, 27, 159
 - for polynomial ring, 24, 279
 - lexicographic, 26, 33, 278
 - partial, 142, 282
 - position-over-term (POT), 27, 31
 - refines weight order, 148
 - reverse lexicographic, 26, 106, 277, 282
 - term-over-position (TOP), 27
- tessellation, 180
- topology
 - relative cellular, 91
 - simplicial, viii, 9
- Tor, 15, 19, 20, 83, 157, 175
- toric ideal, 148, 244, 281, 282, *see also* lattice ideal
- toric variety, 23, 191, 198, 200–208, 246
 - affine, 193–194, 196, 200, 201
 - as sagbi degeneration, 281
 - as spector, 202
 - determined by equivalent data, 202
 - diagonal embedding, 207
 - from polytope, 197–198, 202–203, 205, 207
 - from quiver, 197, 208
 - projective, 71, 195–198
 - smooth, 196, 207
- torus
 - algebraic, *see* algebraic torus
 - as hull complex, 186
- totally ordered group, 133, 152
- translate
 - A -graded, 153, 174
 - \mathbb{N}^n -graded, 6, 15
 - \mathbb{Z}^d -graded, 269
 - \mathbb{Z}^n -graded, 177, 261
- transposition, *see* adjacent transposition
- tree, 69, 197
- triangle, lattice point-free, 147
- triangulation, 50, 77, 148, *see also* subdivision

- as Scarf complex, 114
- in polynomial time, 243
- of polytope, 377
- of saturated semigroup, 243
- regular, 113, 114, 123, 144, 146, 148, 232
- twisted cubic, 169, 170, 171, 174
- stick, *see* stick twisted cubic

- underlying cell complex, 92
- unit
 - in Laurent series ring, 154
 - in semigroup, 133, 151
 - in semigroup ring, 133
- universal cover, 183
- universal grading, 172, 308
- upper triangular matrices, *see* Borel group
- Upper Bound Theorem, 19, 119, 266
- upper-semicontinuity, 160, 172

- variety
 - affine, 195
 - cohomology over quotient, 256
 - degeneration of, 311
 - determinantal, *see* determinantal variety
 - irreducible, 193, 295, 301, 342
 - morphism of, *see* morphism
 - of 2×2 minors in 2×3 matrix, 206
 - of complexes, 351
 - of irrelevant ideal, 200, 205
 - of linear maps, 331
 - of quiver representations, 332
 - of sequences of linear maps, 331
 - projective, 195
 - quasiprojective, 358
 - smooth, *see* smooth point
 - toric, *see* toric variety
- vector bundle, 309, 353, 367
- vector label, 217
- Veronese subring, 194, 205
- vertex
 - axial, 99
 - of simplicial complex, 4
- vertex denominator, 237
- vertex figure, 105
- vertex K -polynomial, 237
- vertex label, 62
- vertex semigroup, 237
- void complex, 4, 10, 121, 235

- weak order, 298–299, 300, 302
 - induction on, 304, 306
 - on symmetric group, 309, 330
- weight
 - exponential, 307, 344
 - of a term, 142, 158
 - ordinary, 344, 345, 346, 348
 - weight order, 142, 158, 159, 330
 - weight vector, 142, 148, 158, 172
 - generic for I , 142
- word, 326

- Young tableau, *see* tableau
- Young, Alfred, 288

- \mathbb{Z} -grading, *see* standard \mathbb{Z} -grading
- Zariski closed, 25, 358
- Zariski dense, 342
- Zariski open, 25, 358
- Zariski topology, 193
- \mathbb{Z}^d -graded product, 219
- \mathbb{Z}^d -grading, 269
- Zelevinsky map, 331, 334, 338, 341, 352
 - multigrading on, 345
- Zelevinsky permutation, 337, 338–340, 348–350, 352
 - of minimal length (v_*), 338, 349
- zero set, 6
- zero-fiber of Hilb, 367, 368, 378
- zerodivisor, 264, 269
 - in semigroup ring, 131, 132
- Zorn's Lemma, 221