LOSSLESS IMAGE COMPRESSION USING WAVELET TRANSFORMS THAT MAP INTEGERS TO INTEGERS

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ABSTRACT

Invertible wavelet transforms that map integers to integers have important applications in lossless representation. In this paper, we present an approach to build integer to integer wavelet transforms based upon the idea of factoring wavelet transforms into lifting steps. This allows the construction of an integer version of every wavelet transform. We use these transforms for lossless image compression, and demonstrate their effectiveness.

1 INTRODUCTIONS

High-fidelity images generated from studio-quality high-definition video, medical images, seismic data, satellite images, and high-quality images scanned from manuscripts typically demand lossless encoding. Yet, the huge datasize prohibits fast distribution of data. There is thus a need to seek encoding methods that can support storage and transmission of images at a spectrum of resolutions and encoding fidelities, from lossy to lossless, for progressive delivery and for different end-users' needs.

In recent years, wavelet transforms have been successfully used for lossy encoding of images. The multiresolution nature of the transform is also ideal for progressive transmission. However, the wavelet filters that are used have floating point coefficients. Thus, when the input data consist of sequences of integers (as is the case for images), the resulting filtered outputs no longer consist of integers. Yet, for lossless encoding, it would be of interest to be able to characterize the output completely again with integers.

We denote by $(s_{0,j})_j$ the original signal of interest, $(s_{1,j})_j$ and $(d_{1,j})_j$ the lowpass and highpass coefficients respectively after a wavelet transform. In this paper, we present constructions of wavelet transforms that map a signal $(s_{0,j})_j$ represented in integers to $(s_{1,j})_j$ and $(d_{1,j})_j$, also represented in integers. The transform is reversible, i.e., we can exactly recover $(s_{0,j})_j$ from $(s_{1,j})_j$ and $(d_{1,j})_j$. We also show applications of the proposed transforms for lossless image compression.

One example wavelet transform that maps integers to integers is the Haar transform, when written in its unnormalized form:

$$\begin{aligned} d_{1,l} &= s_{0,2l+1} - s_{0,2l} \\ s_{1,l} &= s_{1,l} - \lfloor d_{1,l}/2 \rfloor . \end{aligned} \quad (1)$$

This form is known as the S transform [1]. Said and Pearlman [2] further proposed the S+P transform (**S** transform + **P**rediction) in which linear prediction is performed on the lowpass coefficients $s_{1,l}$ to generate a new set of highpass coefficients after an S transform. The general form of the transform is:

$$\begin{aligned} d_{1,l}^{(1)} &= s_{0,2l+1} - s_{0,2l} \\ s_{1,l} &= s_{0,2l} + \lfloor d_{1,l}^{(1)}/2 \rfloor \\ d_{1,l} &= d_{1,l}^{(1)} + \lfloor \alpha_{-1}\,(s_{1,l-2} - s_{1,l-1}) + \alpha_0\,(s_{1,l-1} - s_{1,l}) \\ &\quad +\alpha_1\,(s_{1,l} - s_{1,l+1}) - \beta_1\, d_{1,l+1}^{(1)} \rfloor . \end{aligned} \quad (2)$$

The TS-transform proposed in [3] is a special case, when $\alpha_{-1} = \beta_1 = 0$ and $\alpha_0 = \alpha_1 = 1/4$. It is an integer version of the $(3,1)$ biorthogonal wavelet transform of Cohen-Daubechies-Feauveau [4]. Said and Pearlman examine several choices for (α_w, β_1) and in the case of natural images suggest $\alpha_{-1} = 0, \alpha_0 = 2/8, \alpha_1 = 3/8$ and $\beta_1 = -2/8$. It is interesting to note that, even though this was not their motivations, this choice without truncation yields a high pass analyzing filter with 2 vanishing moments. The S, TS and S+P transforms can all be seen as special cases of the transforms we propose in this paper.

2 WAVELET TRANSFORMS THAT MAP INTEGERS TO INTEGERS

In this section, we describe constructions of wavelet transforms that map integers to integers. The readers are referred to [5] for more details. The construction is based on writing a wavelet transform in terms of lifting [6], which is a flexible technique that has been applied to the construction of wavelets through an iterative process of updating a subband from an appropriate linear combination of the other subband.

Table 1 lists example wavelet transforms implemented as invertible integer wavelet transforms. The first set of transforms have names of the form $(N, \tilde{N})$, where N is the number of vanishing moments of the analyzing high pass filter, while $\tilde{N}$ is the number of vanishing moments of the synthesizing high pass filter. They are instances of a family of symmetric, biorthogonal wavelet transforms built from the interpolating Deslauriers-Dubuc scaling functions [6]. The next transform $(2+2,2)$ is inspired by the S+P transform — we use one extra lifting step to build the earlier $(2,2)$ into a transform with 4 vanishing moments of the high pass analyzing filter. The idea is to first compute a $(2,2)$ yielding low pass samples $s_{1,l}$ and preliminary detail or high pass samples $d_{1,l}^{(1)}$,

Name	Wavelet Transform	Remarks
(2,2)	$d_{1,l} = s_{0,2l+1} - \lfloor 1/2\,(s_{0,2l} + s_{0,2l+2}) + 1/2 \rfloor$ $s_{1,l} = s_{0,2l} + \lfloor 1/4(d_{1,l-1} + d_{1,l}) + 1/2 \rfloor$	symmetric biorthogonal interpolating
(4,2)	$d_{1,l} = s_{0,2l+1} - \lfloor 9/16\,(s_{0,2l} + s_{0,2l+2}) - 1/16\,(s_{0,2l-2} + s_{0,2l+4}) + 1/2 \rfloor$ $s_{1,l} = s_{0,2l} + \lfloor 1/4(d_{1,l-1} + d_{1,l}) + 1/2 \rfloor$	
(6,2)	$d_{1,l} = s_{0,2l+1} - \lfloor 75/128\,(s_{0,2l} + s_{0,2l+2}) - 25/256\,(s_{0,2l-2} + s_{0,2l+4})$ $+3/256\,(s_{0,2l-4} + s_{0,2l+6}) + 1/2 \rfloor$ $s_{1,l} = s_{0,2l} + \lfloor 1/4(d_{1,l-1} + d_{1,l}) + 1/2 \rfloor$	$K = 1$
(4,4)	$d_{1,l} = s_{0,2l+1} - \lfloor 9/16\,(s_{0,2l} + s_{0,2l+2}) - 1/16\,(s_{0,2l-2} + s_{0,2l+4}) + 1/2 \rfloor$ $s_{1,l} = s_{0,2l} + \lfloor 9/32\,(d_{1,l-1} + d_{1,l}) - 1/32\,(d_{1,l-2} + d_{1,l+1}) + 1/2 \rfloor$	
(2+2,2)	$d^{(1)}_{1,l} = s_{0,2l+1} - \lfloor 1/2(s_{0,2l} + s_{0,2l+2}) + 1/2 \rfloor$ $s_{1,l} = s_{0,2l} + \lfloor 1/4(d^{(1)}_{1,l-1} + d^{(1)}_{1,l}) + 1/2 \rfloor$ $d_{1,l} = d^{(1)}_{1,l} - \lfloor \alpha\,(-1/2\,s_{1,l-1} + s_{1,l} - 1/2\,s_{1,l+1})$ $+\beta\,(-1/2\,s_{1,l} + s_{1,l+1} - 1/2\,s_{1,l+2}) + \gamma\,d^{(1)}_{1,l+1} + 1/2 \rfloor$	$\alpha = \beta = 1/8, \gamma = 0$ 4 vanishing moments of high pass filter K = 1
D4	$d^{(1)}_{l} = s_{0,2l+1} - \lfloor \sqrt{3}\,s_{0,2l} + 1/2 \rfloor$ $s_{1,l} = s_{0,2l} + \lfloor \sqrt{3}/4\,d^{(1)}_{1,l} + (\sqrt{3} - 2)/4\,d^{(1)}_{1,l-1} + 1/2 \rfloor$ $d_{1,l} = d^{(1)}_{1,l} + s_{1,l+1}$ $K = (\sqrt{3} + 1)/\sqrt{2} \approx 1.577$	orthogonal
(9-7)	$d^{(1)}_{1,l} = s_{0,2l+1} + \lfloor \alpha\,(s_{1,2l} + s_{1,2l+2}) + 1/2 \rfloor$ $s^{(1)}_{1,l} = s_{0,2l} + \lfloor \beta\,(d^{(1)}_{1,l} + d^{(1)}_{1,l-1}) + 1/2 \rfloor$ $d_{1,l} = d^{(1)}_{1,l} + \lfloor \gamma\,(s^{(1)}_{1,l} + s^{(1)}_{1,l+1}) + 1/2 \rfloor$ $s_{1,l} = s^{(1)}_{1,l} + \lfloor \delta\,(d_{1,l} + d_{1,l-1}) + 1/2 \rfloor$ $K \approx 1.1496$	symmetric biorthogonal $\alpha \approx -1.586$ $\beta \approx -0.053$ $\gamma \approx 0.883$ $\delta \approx 0.444$

Table 1: Wavelet transforms implemented as invertible integer wavelet transforms

and then use the $s_{1,l}$ combined with $d_{1,l+n}$ $(n > 0)$ to compute $d'_{1,l}$ as a prediction for $d^{(1)}_{1,l}$. The final detail sample then is $d^{(1)}_{1,l} - d'_{1,l}$. Without truncation, we want the scheme to have 4 vanishing moments. This leads to 2 linear equations in 3 unknowns. Special cases are: (1) $\alpha = 1/6, \beta = 0, \gamma = 1/3$, (2) $\alpha = 1/8, \beta = 1/8, \gamma = 0$, and (3) $\alpha = 1/4, \beta = -1/4, \gamma = 1$. In our experiments we found that (2) works considerably better than (1) and (3), and this is the case we use when we refer to $(2+2,2)$ in Section 3. The next two transforms D4 and (9-7) [8] follow from the lifting factorization in [7]. There is further a need for a final scaling step (multiplying the lowpass coefficients by $1/K$ and highpass coefficients by K); the multiplications can be implemented as three extra lifting steps [7].

3 APPLICATION TO LOSSLESS IMAGE COMPRESSION

We apply the various wavelets described in the previous section for lossless compression of digital images. For the evaluation, we use selected images from the set of standard ISO test images [9]. In this set of test images, there are natural images, computer-generated images, compound images (mixed texts and natural images, e.g., "cmpnd1" and "cmpnd2"; "us" is an ultrasound image with text on it) and different types of medical images ("x_ray", "cr", "ct", and "mri"). Separable two dimensional wavelet transforms are taken of the images.

The effectiveness for lossless compression is measured using the first order entropy. We further take into account the fact that the statistics in different quadrants of a wavelet-transformed image are different, and compute the weighted mean of the entropies in each quadrant of the transformed image. In the evaluation, we decompose each image into a maximum of five scales, the length and width permitting. The resulting bit rates are tabulated in Table 2.

We note that there is no filter that consistently performs better than all other filters on the test images. Wavelet filters with more analyzing vanishing moments generally perform well with natural and smooth images and not so with images with a lot of edges and high frequency components, such as with compound images. On the other hand, a low order filter like S-transform generally performs the worst, especially with natural images. It does a poor job in decorrelating the images. However, it performs significantly better on compound images (e.g., "cmpnd1" and "cmpnd2") than other higher order filters. Filters (4,2) and (2+2,2) have similar performances and generally perform better than other filters evaluated. For images "finger" and "ct", they perform significantly better than other filters. It is interesting to note that even though the S+P has 2 analyzing vanishing moments, it performs better than the TS which has 3 and has comparable performances with those with 4. This suggests that there are other factors besides the number of analyzing vanishing moments which affect compression efficiency. Furthermore, the $(9-7)$, which is most popularly used for lossy compression of images and which has 4 analyzing vanishing moments, generally does not perform as well as the (4,2) and (2+2,2), which have the same number of analyzing vanishing moments.

We have also evaluated the wavelet filters by attaching an entropy coder to compute the *actual* bit rate. The results will be reported in the final paper. We found that the relative strength of each filter over the others in actual coding is similar to that computed using the entropies. In actual coding, wavelet filters (4,2) and (2+2,2) generally

Images	S	TS	(2,2)	S+P	(4,2)	(2+2,2)	$D4$	9-7
air2	5.13	4.91	4.82	4.81	4.77	**4.76**	5.09	4.98
bike	4.36	4.28	4.19	**4.18**	4.20	4.21	4.37	4.31
cafe	5.69	5.54	**5.41**	5.42	**5.41**	**5.41**	5.63	5.51
cats	3.69	3.53	3.47	3.43	**3.42**	**3.42**	3.60	3.47
cmpnd1	**2.25**	2.84	2.79	2.97	3.31	3.36	3.04	3.45
cmpnd2	**2.41**	2.96	2.79	3.01	3.28	3.33	3.12	3.42
cr	5.40	5.25	**5.20**	5.24	5.22	5.22	5.28	5.22
ct	5.54	4.63	4.50	4.30	**4.15**	4.16	4.96	4.36
faxballs	1.61	1.31	**1.08**	1.41	1.36	1.17	1.54	1.97
finger	6.24	5.71	5.49	5.48	**5.35**	**5.35**	5.85	5.45
gold	4.27	4.10	4.05	4.08	4.04	**4.03**	4.19	4.14
graphic	3.18	2.82	2.60	2.67	**2.56**	**2.56**	3.08	3.00
hotel	4.30	4.18	**4.03**	4.10	4.06	4.04	4.25	4.18
mri	6.59	6.16	6.02	**5.90**	5.91	5.91	6.26	5.97
tools	5.84	5.80	**5.69**	5.73	5.73	5.72	5.88	5.81
us	**3.64**	3.79	3.69	3.79	3.87	3.85	3.95	4.26
water	2.46	2.45	**2.42**	2.47	2.45	2.44	2.46	2.50
woman	4.87	4.67	4.57	4.54	**4.53**	4.54	4.78	4.64
x_ray	6.42	6.13	**6.06**	6.09	**6.06**	**6.06**	6.18	6.08

Table 2: Bit rates of transformed images in entropies.

outperform other filters for natural and smooth images. Also, filters with more analyzing vanishing moments perform poorly with compound images. We also found that numbers computed from entropy computations provide a good indication of the actual performance of a wavelet filter.

The big advantage of using wavelet transform to represent images is multiresolution representation, which lossless compression methods based on spatial-domain prediction (see [10] and the references therein for comparisons of state-of-the-art spatial-domain predication techniques) cannot offer. Using wavelet transforms that map integers to integers permits lossless representation of the image pixels and easily allows the transmission of lower resolution versions first, followed by transmissions of successive details. Such a mode of transmission is especially valuable in scenarios where bandwidth is limited, image sizes are large and lossy compression is not desirable. Examples are transmission of 2-D and 3-D medical images for telemedicine applications and transmission of satellite images down to earth.

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