Math 4108 Homework 14
Due at the beginning of class on Tuesday, April 21.

§13.6 #7
§13.7 #1

Not from the text:

1. (a) Let A be a fractional ideal in $\mathbf{Q}(\delta)$ and let $\overline{A} = \{\overline{\alpha} \,:\, \alpha \in A\}$. Prove that $A\overline{A}$ is a principal fractional ideal.
   (b) Let A be a fractional ideal in $\mathbf{Q}(\delta)$. Prove that $\langle A\rangle^{-1} = \langle \overline{A}\rangle$.
   (c) Prove that two fractional ideals A, A' in $\mathbf{Q}(\delta)$ are similar if and only if there exists a fractional ideal C such that AC and $A'C$ are both principal.
2. (a) Prove that the quadratic integer ring in $\mathbf{Q}(\sqrt{-7})$ is a PID. [Hint: what is $\lfloor \mu \rfloor$ in this case?]
   (b) Do the same for $\mathbf{Q}(\sqrt{-11})$.
3. Let $d = -163$, $\delta = \sqrt{d}$, and $\eta = \frac{1}{2}(1+\delta)$.
   (a) Prove that $\mathrm{Cl}(\mathbf{Q}(\delta))$ is generated by the classes of the prime ideals P with $N(P) \in \{2, 3, 5, 7\}$.
   (b) Prove that $\mathbf{Z}[\eta]$ is a PID.
4. Prove that the quadratic integer rings in $\mathbf{Q}(\sqrt{-19})$, $\mathbf{Q}(\sqrt{-43})$, and $\mathbf{Q}(\sqrt{-67})$ are all PIDs. This completes Gauss' list: you've proved on the homework that the quadratic integer ring in $\mathbf{Q}(\sqrt{d})$ is a PID for $d = -2, -3, -7, -11, -19, -43, -67, -163$. [We did $d = -1$ in class.]
5. Let $d = -6$ and $\delta = \sqrt{d}$.
   (a) Factor the ideals (2) and (3) into prime ideals in $\mathbf{Z}[\delta]$.
   (b) Factor the ideal (6) into prime ideals in $\mathbf{Z}[\delta]$.
   (c) Determine the class group of $\mathbf{Q}(\delta)$.