Math 4803/8803 Homework 1
Due at the beginning of class on Wednesday, August 26.

You are encouraged to collaborate on homework assignments! Just remember to write up your proofs separately and to acknowledge your collaborators on your work.

The exercises in Samuel are found at the back of the book. Please complete:
Chapter I #2.

Exercises not from the text:

(1) Let A be an integral domain. Prove that $A[\![X_1, \ldots, X_n]\!]^\times$ consists of all power series whose constant coefficient is a unit in A.
(2) Let K be a field.
   (a) Prove that every nonzero ideal in $K[\![X]\!]$ is generated by a power of X. Hence $K[\![X]\!]$ is a principal ideal domain.
   (b) Let $K(\!(X)\!)$ be the fraction field of $K[\![X]\!]$. Which common group is $K(\!(X)\!)^\times / K[\![X]\!]^\times$ isomorphic to? Give an explicit isomorphism.
(3) Let A be a principal ideal domain with fraction field K and let $x, y \in K^\times$. Prove that $\operatorname{lcm}(x, y) \gcd(x, y) = xy$ (up to associates).