MATH 4803/8803: INTRODUCTION TO ALGEBRAIC NUMBER THEORY
FALL 2015 SYLLABUS

Instructor:	Joe Rabinoff
Time:	10:05–10:55am, MWF
Location:	**Skiles 005** (this is not the room listed on Oscar)
Course Website:	http://people.math.gatech.edu/~jrabinoff/1516F-4803/
Prerequisite:	Math 4108 or Math 6121
Text:	P. Samuel, *Algebraic Theory of Numbers* (English edition)
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Office:	Skiles 221
Office hours:	Mondays, 2–3pm; Tuesdays, 1–3pm; Wednesdays, 12–1pm; and by appointment

Objectives. This course is an introduction to the core concepts of algebraic number theory: number fields, Dedekind domains, unique factorization of ideals, the ideal class group, and Dirichlet's unit theorem. These tools provide solutions to several problems which are elementary to state but surprisingly difficult to resolve, including Pell's equation $x^2 - Dy^2 = 1$, quadratic reciprocity, the two squares theorem (classifying which prime numbers arise as a sum of two square integers), the four squares theorem (every positive integer is a sum of four square integers), and Kummer's proof of Fermat's last theorem on nonexistence of $x, y, z \in \mathbf{Z}$ with $xyz \neq 0$ and $x^p + y^p = z^p$, in the case of a "regular" prime exponent p. I will try to present the material as much as possible in the context of these motivating questions.

This is a course in pure mathematics. The material has applications in computer science, especially cryptography, but such applications will not feature in this course. After taking this course, students will be prepared to take a graduate course in algebraic number theory or basic class field theory, and will have more than sufficient mathematical background to learn cryptography.

I would like to be explicit about the following points:

- This course is intended for students with a fair amount of experience with abstract algebra. We will not spend time reviewing basic facts about groups, rings or fields. Students are expected to have a working understanding of finite fields, principal ideal domains, etc. All of the work on which you will be evaluated will be some form of written proof.
- The only way to learn abstract math involves investing a lot of time outside of class, every single week. I can give you a trail map, point out the nice views, and warn you of any pitfalls, but only you can hike to the top of the mountain. In other words, you need to think hard and you need a long attention span. If you are taking six other classes, this is probably not the class for you.

Homework.

- The weekly homework is the *most important component* of the course; as such, it will be heavily emphasized in the grading scheme (below).
- There will generally be a homework assignment due every Wednesday. **This includes the Wednesday during the week before finals period.**
- Late homework will generally not be accepted.
- All solutions must be neatly written in grammatically correct mathematical English.

- Collaboration on solving homework problems is **encouraged**, and may be necessary; however, **you must write up your work separately**, so your proofs will not be identical word-for-word.
- Please list your collaborators and any outside sources you consulted on all graded work. This is matter of academic honesty: standing on the shoulders of giants (or classmates) is fine, but you cannot tacitly take credit for other peoples' ideas.
- I'd like to discourage you from searching the Internet for help. The problems are assigned so that you spend the necessary time thinking about the material: you learn when you get stuck and think hard for a long time. Circumventing this process is shooting yourself in the foot.
- I expect that the homework assignments will take **about 6 hours** to complete each week (although any given student may require more or less time than that). I do not expect you to be able to complete them in a satisfactory way the night before they are due. I hope that you will start them the weekend before, so that you can discuss the more difficult problems with each other and with me during office hours early in the week.
- If you start the homework early, collaborate with your classmates, talk to me in office hours, and commit the necessary time, there is no reason you should do poorly on the homework.

Exams and project.

- There will be one in-class midterm exam.
- There will be no aids (textbook, calculator, etc.) allowed on the midterm.
- Absences from the midterm are generally excused only for Georgia Tech official business, religious holidays, serious illness, and the like. This does not include internships and interviews. I reserve the right to ask for a letter from the Dean of Students to justify an absence.
- In the event that you miss the midterm for an acceptable reason, you will be excused from that exam; the homework and final will count more towards your grade.
- Students will have the option to complete either a take-home final exam or a final project.
- The final exam, completed during finals week, will be open-note and open-book, and should be similar in length and difficulty to a weekly homework assignment, although it covers the material from the entire course. Collaboration is **not** allowed on the final.
- The final project consists of a 3–5 page expository account of a subject which is related to but not directly covered in the lecture. A good example would be Kummer's complete proof of Fermat's Last Theorem for regular prime exponents, which we will only sketch in class if we get to it. The project will take longer to complete than the final, but you can start it whenever you like and discuss it with me and your classmates as much as you want.

Lectures. The course is organized under the assumption that students will attend all of the lectures. If you are obliged to be absent, please let me know so that I can catch you up on anything you may have missed, such as announcements or material not contained in the book.

Honor code. Students are expected to fully adhere to the honor code, which can be found at

http://www.honor.gatech.edu/

Note that writing up your homework separately and listing your collaborators are matters of academic honesty.

Important dates. The midterm date is *tentative*; it will be confirmed at least a week in advance.

October 5	Midterm exam
October 25	Last day to withdraw from course
December 7–11	Final exam period

Grading. The grade breakdown for the course is as follows:

40%	Homework assignments
25%	Midterm exam
35%	Final exam / final project

The final letter grade cutoffs will be determined *after* all number grades have been determined. However, if you score at least 90% in the class you will be guaranteed an A, 80% for a B, and 70% for a C.

Outline. Below is a rough[1] course schedule with corresponding sections of the text. We will follow the text rather closely, with occasional deviations and additions.

August 17 – September 18 (14 lectures): Chapters I and II: divisibility; modules over a PID; integral closure; norm and trace; the discriminant; cyclotomic fields.

September 21 – October 2 (6 lectures): Chapter III: Noetherian rings; Dedekind rings; unique factorization of ideals; the ideal class group; applications

October 7 – November 11 (15 lectures): Chapters IV and V: geometry of numbers; finiteness of the ideal class group; Dirichlet's unit theorem; Pell's equation; splitting of prime ideals; ramification; quadratic reciprocity; two and four square theorems.

November 13 – December 4 (8 lectures): Other topics, depending on time and interest: relations to Galois theory; sketch of Kummer's proof of Fermat's last theorem for regular prime exponents; introduction to class field theory; overflow material from Chapters I–V.

[1] This is my first time teaching this course, so I can only estimate how long each topic will take.