MATH 6421: ALGEBRAIC GEOMETRY I

Instructor:	Joe Rabinoff
Time:	11:05–11:55am, MWF
Location:	Skiles 246
Course Website:	http://people.math.gatech.edu/~jrabinoff/1516F-6421/
Prerequisite:	Abstract algebra (4107/4108 or 6121)
Texts:	See the links on the website.
Email:	rabinoff@math.gatech.edu
Office:	Skiles 221
Office hours:	Mondays, 2–3pm; Tuesdays, 1–3pm; Wednesdays, 12–1pm; and by appointment

Objectives. This course is an introduction to the basic principles of algebraic geometry. Very roughly, algebraic geometry is the study of the geometric properties of the set of simultaneous zeros of some number of polynomial equations. The field of algebraic geometry is huge, thriving, and extremely well-developed. Unfortunately, the foundational component of the modern theory (the language of schemes) requires well over one semester of graduate study to absorb, and demands a fluency with commutative algebra, complex analysis, and complex manifolds that reaches far beyond the prerequisites for this course.

All of these considerations make designing a first course in algebraic geometry not a straightforward task. Presenting the foundational material in a systematic way would be extremely formal and dry, and would take far longer than one semester to complete. Therefore some compromises have to be made in order to teach an interesting and self-contained course. In this class, you will primarily learn the classical (19th century) theory of varieties over an algebraically closed field. When possible, I will present the theory from a more modern viewpoint, and I will attempt to give the ideas behind the more general parallel theory of schemes, showing how they arise naturally as elegant solutions to certain awkward problems with the classical theory. Topics include affine varieties, the Zariski topology, morphisms and the sheaf of regular functions, projective varieties, smooth varieties. This part of the course will mostly follow Gathmann's notes.

We will then study the babymost case[1] of varieties in detail: the theory of smooth algebraic curves, or varieties of dimension 1. This is a very well-understood and beautiful theory which has applications throughout mathematics: for instance, over the complex numbers it is precisely equivalent to the analytic theory of compact Riemann surfaces, and in genus 1 it is the study of elliptic curves, which play a central role in number theory and cryptography. Topics include the Galois correspondence between function fields and curves, plane curves of degrees 2, 3, and 4, elliptic curves, the Riemann–Hurwitz formula, and potentially the Riemann–Roch formula. This part of the course will mostly follow Fulton's book.

Homework.

- There will generally be a homework assignment due every Friday. **This includes the Friday during the week before finals period.**
- Algebraic geometry is not a subject you can learn in any meaningful way just by sitting in on a class. You have to do the homework assignments, which will take many (hopefully interesting

[1] as Nick Katz would say

and fun) hours to complete. I expect that you will not do well on the homework assignments if you start them on Thursday night.
- Your homework scores determine a majority of your final grade.
- Late homework will generally not be accepted.
- Collaboration on solving homework problems is **encouraged**, and may be necessary; however, **you must write up your work separately**, so your proofs will not be identical word-for-word.
- Please list your collaborators and any outside sources you consulted on all graded work.
- If you start the homework early, collaborate with your classmates, talk to me in office hours, and commit the necessary time, there is no reason you should do poorly on the homework.

Final project. There will be no exams. However, you will be required to write an expository 5–10 page final paper on a topic of interest to be chosen with the help of the instructor.

Honor code. Students are expected to fully adhere to the honor code, which can be found at

`http://www.honor.gatech.edu/`

Note that writing up your homework separately and listing your collaborators are matters of academic honesty.

Grading. The grade breakdown for the course is as follows:

60% Homework assignments
40% Final paper

The final letter grade cutoffs will be determined *after* all number grades have been determined. However, if you score at least 90% in the class you will be guaranteed an A, 80% for a B, and 70% for a C.