Welcome!

- The syllabus and course materials are on my website:

  `http://people.math.gatech.edu/~jrabinoff/1617F-1553/`

  (There is a link from T-Square.) **Read it!** Chances are, it answers all your administrative questions.
- There is an easy one-problem homework set called *Warmup* on WebWorK whose sole purpose is to make sure you're able to login to the system and successfully get credit for your answers. It is due Friday.
- Enroll in Piazza (the link is on T-Square). You can ask questions there, and we will use it for in-class polling on a daily basis. I will also send announcements through Piazza, and you won't receive them until you enroll. **Please use your T-Square email address to enroll**, so that your poll responses show up in the T-Square gradebook.
- Bring your smartphone or laptop to class, but please don't use it unless we're actually doing a poll.
- My office is Skiles 221 and my office hours are Wednesday, 1–2pm and Thursday, 3:30–4:30pm.

Math 1553
Introduction to Linear Algebra

School of Mathematics
Georgia Institute of Technology

Introduction to Linear Algebra

Linear. Algebra.

What is Linear Algebra?

Linear

- having to do with lines/planes/etc.
- For example, $x + y + 3z = 7$, not $\sin, \log, x^2$, etc.

Algebra

- solving equations involving numbers and symbols
- from al-jebr (Arabic), meaning reunion of broken parts
- 9^{th} century Abu Ja'far Muhammad ibn Muso al-Khwarizmi

Why a whole course?

But these are the easiest kind of equations! I learned how to solve them in 7th grade!

Ah, but engineers need to solve *lots* of equations in *lots* of variables.

$$\begin{aligned}
3x_1 + 4x_2 + 10x_3 + 19x_4 - 2x_5 - 3x_6 &= 141 \\
7x_1 + 2x_2 - 13x_3 - 7x_4 + 21x_5 + 8x_6 &= 2567 \\
-x_1 + 9x_2 + \tfrac{3}{2}x_3 + x_4 + 14x_5 + 27x_6 &= 26 \\
\tfrac{1}{2}x_1 + 4x_2 + 10x_3 + 11x_4 + 2x_5 + x_6 &= -15
\end{aligned}$$

Often, it's enough to know some information about the set of solutions without having to solve the equations at all!

Also, what if one of the coefficients of the x_i is itself a parameter — like an unknown real number t?

Linear Algebra in Engineering

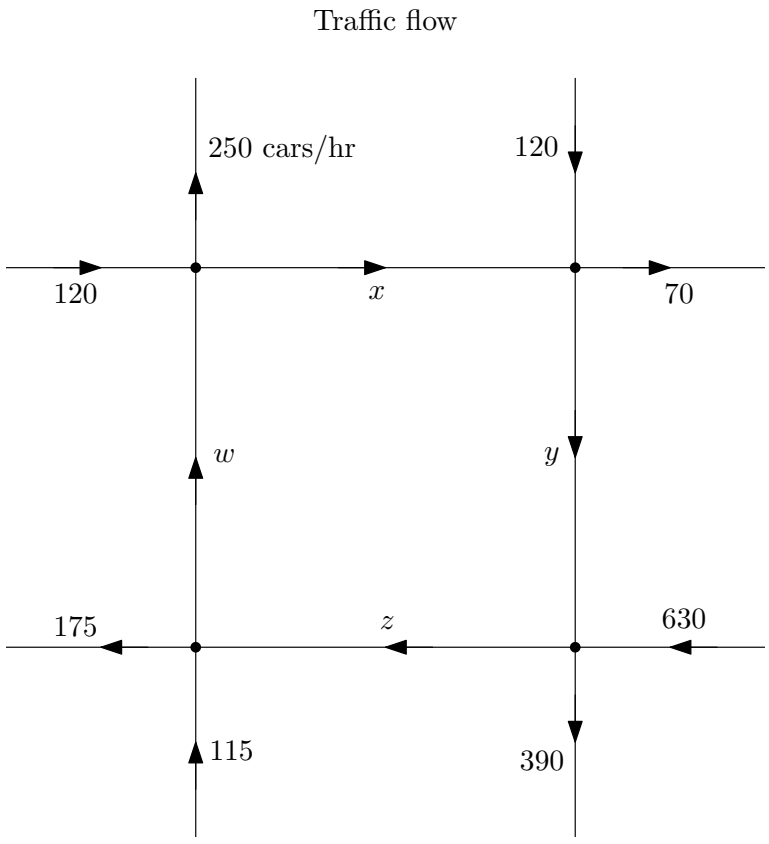
Large classes of engineering problems, no matter how huge, can be reduced to linear algebra:

$$Ax = b \quad \text{or}$$
$$Ax = \lambda x$$

"...and now it's just linear algebra"

Applications of Linear Algebra

Civil Engineering: How much traffic flows through the four labeled segments?



$\rightsquigarrow$ system of linear equations:

$$w + 120 = x + 250$$
$$x + 120 = y + 70$$
$$y + 630 = z + 390$$
$$z + 115 = w + 175$$

Applications of Linear Algebra

Chemistry: Balancing reaction equations

$$\_\_\_\ C_2H_6 + \_\_\_\ O_2 \to \_\_\_\ CO_2 + \_\_\_\ H_2O$$

$\rightsquigarrow$ system of linear equations, one equation for each element.

$$\begin{aligned} 2x &= z \\ 6x &= 2w \\ 2y &= 2z \end{aligned}$$

Applications of Linear Algebra

Biology: In a population of rabbits...

- half of the new born rabbits survive their first year
- of those, half survive their second year
- the maximum life span is three years
- rabbits produce 0, 6, 8 rabbits in their first, second, and third years

If I know the population in 2016 (in terms of the number of first, second, and third year rabbits), then what is the population in 2017? $\rightsquigarrow$ system of linear equations:

$$\begin{aligned} 6y_{2016} + 8z_{2016} &= x_{2017} \\ \tfrac{1}{2}x_{2016} &= y_{2017} \\ \tfrac{1}{2}y_{2016} &= z_{2017} \end{aligned}$$

Question

Does the rabbit population have an asymptotic behavior? Is this even a linear algebra question? Yes, it is!

Applications of Linear Algebra

Geometry and Astronomy: Find the equation of a circle passing through 3 given points, say $(1,0)$, $(0,1)$, and $(1,1)$. The general form of a circle is $a(x^2+y^2)+bx+cy+d=0$. $\rightsquigarrow$ system of linear equations:

$$\begin{aligned} a+b\quad\quad +d &= 0\\ a\quad\quad +c+d &= 0\\ 2a+b+c+d &= 0 \end{aligned}$$

Very similar to: compute the orbit of a planet: $ax^2+by^2+cx+dy+e=0$

Applications of Linear Algebra

Google: "The 25 billion dollar eigenvector." Each web page has some importance, which it shares via outgoing links to other pages $\rightsquigarrow$ system of linear equations.

Larry Page flies around in a private 747 because he paid attention in his linear algebra class!

Stay tuned!

Overview of the course

- Solve the matrix equation $Ax = b$
  - Solve systems of linear equations using matrices, row reduction, and inverses
  - Solve systems of linear equations with varying parameters using parametric forms for solutions, the geometry of linear transformations, the characterizations of invertible matrices, and determinants

- Solve the matrix equation $Ax = \lambda x$
  - Solve eigenvalue problems through the use of the characteristic polynomial
  - Understand the dynamics of a linear transformation via the computation of eigenvalues, eigenvectors, and diagonalization

- Almost solve the equation $Ax = b$
  - Find best-fit solutions to systems of linear equations that have no actual solution using least squares approximations