① What are 3 possible solutions for the reduced row echelon form of the augmented matrix?

② Draw pictures for above 3

Math 1553 Worksheet 2

September 2, 2016

③ Q's? (lin combo?, Span?, Parametric?, Row Reduction alg?, Pivot/free variable/leading meaning, Proof writing?)

1. Find the parametric form for the general solution to the following system of equations, if such a solution exists: (Vectors in $\mathbb{R}^n$)

$$\begin{cases} x_1 + x_2 + x_3 - x_4 = -3 \\ 2x_1 + 3x_2 + x_3 - 5x_4 = -9 \\ x_1 + 3x_2 - x_3 - 6x_4 = 7 \end{cases} \rightsquigarrow (x_1, x_2, x_3, x_4) = (?, ?, ?, ?).$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & -1 & -3 \\ 2 & 3 & 1 & -5 & -9 \\ 1 & 3 & -1 & -6 & 7 \end{array}\right) \xrightarrow[R_3 = R_1 - R_3]{R_2 = R_1 - \frac{1}{2}R_2} \left(\begin{array}{cccc|c} 1 & 1 & 1 & -1 & -3 \\ 0 & -\frac{1}{2} & \frac{1}{2} & \frac{3}{2} & \frac{3}{2} \\ 0 & -2 & 2 & 5 & -10 \end{array}\right) \xrightarrow[R_3 = \frac{1}{2}R_3]{R_2 = -2 \cdot R_2} \left(\begin{array}{cccc|c} 1 & 1 & 1 & -1 & -3 \\ 0 & 1 & -1 & -3 & -3 \\ 0 & -1 & 1 & \frac{5}{2} & -5 \end{array}\right) \xrightarrow{R_3 = R_2 + R_3} \left(\begin{array}{cccc|c} 1 & 1 & 1 & -1 & -3 \\ 0 & 1 & -1 & -3 & -3 \\ 0 & 0 & 0 & -\frac{1}{2} & -8 \end{array}\right)$$

$$\xrightarrow[R_3 = -2R_3]{R_1 = R_1 - R_2} \left(\begin{array}{cccc|c} 1 & 0 & 2 & 2 & 0 \\ 0 & 1 & -1 & -3 & -3 \\ 0 & 0 & 0 & 1 & 16 \end{array}\right) \xrightarrow[R_2 = R_2 + 3R_3]{R_1 = R_1 - 2R_3} \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \left(\begin{array}{cccc|c} 1 & 0 & 2 & 0 & -32 \\ 0 & 1 & -1 & 0 & 45 \\ 0 & 0 & 0 & 1 & 16 \end{array}\right) \end{array} \Rightarrow \begin{array}{l} x_1 + 2x_3 = -32 \\ x_2 - x_3 = 45 \\ x_4 = 16 \end{array} \Rightarrow \begin{array}{l} x_1 = -2x_3 - 32 \\ x_2 = 1x_3 + 45 \\ x_3 = 1x_3 + 0 \\ x_4 = 0x_3 + 16 \end{array} \Rightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 1 \\ 0 \end{pmatrix} x_3 + \begin{pmatrix} -32 \\ 45 \\ 0 \\ 16 \end{pmatrix}$$

RREF

free variable
- any value of x_3 satifies the above 3 equations

2. Write $\begin{pmatrix} 6 \\ 11 \\ 6 \end{pmatrix}$ as a linear combination of the vectors

$$u = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} \quad v = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} \quad w = \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix}.$$

$$c_1\begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} + c_2\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} + c_3\begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 6 \\ 11 \\ 6 \end{pmatrix}$$

if c_1, c_2, c_3 exist such that above sum is consistent, then $\begin{pmatrix} 6 \\ 11 \\ 6 \end{pmatrix}$ is a linear combination of $\vec{u}, \vec{v}$ & $\vec{w}$ (def

$$\left(\begin{array}{ccc|c} 2 & 1 & 3 & 6 \\ 1 & -1 & 2 & 11 \\ 4 & 3 & 5 & 6 \end{array}\right) \xrightarrow{\text{RREF}} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 1 \end{array}\right) \Rightarrow \begin{array}{l} c_1 = 4 \\ c_2 = -5 \\ c_3 = 1 \end{array}$$

so $4\begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} - 5\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} + 1\begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix} \overset{\text{must}}{\checkmark =} \begin{pmatrix} 6 \\ 11 \\ 6 \end{pmatrix}$ & $\begin{pmatrix} 6 \\ 11 \\ 6 \end{pmatrix} \in \text{Span}\{u, v, w\}$

If c_1, c_2, or c_3 were 0, what Span would $\begin{pmatrix} 6 \\ 11 \\ 6 \end{pmatrix}$ be in

3. Decide if each of the following statements is true or false. If it is true, prove it; if it is false, provide a counterexample.

Proof Writing?

a) Every set of four or more vectors in $\mathbf{R}^3$ will span $\mathbf{R}^3$.

$\boxed{\text{False}}$ (ex) $\left\{\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix}\right\} \in \mathbb{R}^3$

these span a line defined by $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} t$

What can we change to make this statement true?

~~Every set~~ $\Rightarrow$ Every set w/ at least 3 linearly independant vectors

- What if there were 4 linearly independant vectors in $\mathbb{R}^3$

b) The span of any set (including the empty set!) contains the zero vector.

$\boxed{\text{True}}$ The span of a set of vectors $\vec{a}_1 \ldots \vec{a}_n$ is all $\vec{b}$ st. $c_1\vec{a}_1 + \ldots + c_2\vec{a}_n = b$ for any values of c_i

In the trivial case, if all c_i's $= 0$, then $\vec{b} = \vec{0}$

If the set is $\{0\}$ then it spans a 0-dimensional space which by convention is the $\vec{0}$ (aka a point)

Therefore, $\vec{0}$ is always in the span of any set.

(2)

$$\left(\begin{array}{ccc|c} 2 & 1 & 3 & 6 \\ 1 & -1 & 2 & 11 \\ 4 & 3 & 5 & 6 \end{array}\right) \xrightarrow[\substack{R_3 = \\ R_1 - \frac{1}{2}R_3}]{\substack{R_2 = \\ R_1 - 2R_2}} \left(\begin{array}{ccc|c} 2 & 1 & 3 & 6 \\ 0 & 3 & -1 & -16 \\ 0 & -1/2 & 1/2 & 3 \end{array}\right) \xrightarrow{6R_3} \left(\begin{array}{ccc|c} 2 & 1 & 3 & 6 \\ 0 & 3 & -1 & -16 \\ 0 & -3 & 3 & 18 \end{array}\right) \xrightarrow{\substack{R_3 = \\ R_2 + R_3}} \left(\begin{array}{ccc|c} 2 & 1 & 3 & 6 \\ 0 & 3 & -1 & -16 \\ 0 & 0 & 2 & 2 \end{array}\right) \xrightarrow[R_3/2]{\substack{R_1/2 \\ R_2/3}} \left(\begin{array}{ccc|c} 1 & 1/2 & 3/2 & 3 \\ 0 & 1 & -1/3 & -16/3 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

$$\xrightarrow[\substack{R_1 = \\ R_1 - \frac{3}{2}R_3}]{\substack{R_2 = \\ R_2 + \frac{1}{3}R_3}} \left(\begin{array}{ccc|c} 1 & 1/2 & 0 & 3/2 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 1 \end{array}\right) \xrightarrow{\substack{R_1 = \\ R_1 - \frac{1}{2}R_2}} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 1 \end{array}\right)$$