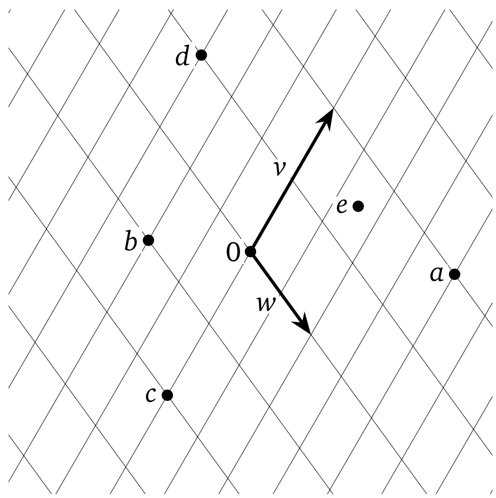
Math 1553 Quiz 2

Solutions

1. [1 point each] Consider the following picture of two vectors v, w:



For each of the labeled points, estimate the coefficients x, y such that the linear combination $xv + yw$ is the vector ending at that point. (Use the parallelogram law for vector addition; you needn't show your work.)

$$\_\_\_\_\_\_ v + \_\_\_\_\_\_ w = a$$

$$\_\_\_\_\_\_ v + \_\_\_\_\_\_ w = b$$

$$\_\_\_\_\_\_ v + \_\_\_\_\_\_ w = c$$

$$\_\_\_\_\_\_ v + \_\_\_\_\_\_ w = d$$

$$\_\_\_\_\_\_ v + \_\_\_\_\_\_ w = e$$

Solution.

As you can tell from the grid, you reach a by following v once then w twice. Hence $a = v + 2w$. Similarly, $b = -\frac{1}{2}v - w$, $c = -v + 0w$, $d = \frac{1}{2}v - \frac{3}{2}w$, and $e = \frac{3}{4}v + \frac{3}{4}w$.

[over]

2. [2 points each] Which of the following vectors are in the span of $\begin{pmatrix}1\\-1\\2\end{pmatrix}$ and $\begin{pmatrix}6\\2\\-1\end{pmatrix}$?

$$\textbf{a)}\ \begin{pmatrix}-14\\-10\\11\end{pmatrix} \qquad \textbf{b)}\ \begin{pmatrix}-14\\-10\\-11\end{pmatrix} \qquad \textbf{c)}\ \begin{pmatrix}0\\0\\0\end{pmatrix}$$

(Show at least a bit of your work.)

Solution.

a) Deciding if $\begin{pmatrix}-14\\-10\\11\end{pmatrix}$ is a linear combination of $\begin{pmatrix}1\\-1\\2\end{pmatrix}$ and $\begin{pmatrix}6\\2\\-1\end{pmatrix}$ amounts to row reducing the augmented matrix:

$$\left(\begin{array}{rr|r}1&6&-14\\-1&2&-10\\2&-1&11\end{array}\right) \overset{\text{row reduce}}{\rightsquigarrow} \left(\begin{array}{rr|r}1&6&-14\\0&1&3\\0&0&0\end{array}\right).$$

We can stop here, because we can already see that the system is consistent. (You weren't asked to find the coefficients in a linear combination.)

b) As before, we row reduce

$$\left(\begin{array}{rr|r}1&6&-14\\-1&2&-10\\2&-1&-11\end{array}\right) \overset{\text{row reduce}}{\rightsquigarrow} \left(\begin{array}{rr|r}1&6&-14\\0&1&3\\0&0&-22\end{array}\right).$$

We can stop here, because this corresponds to an inconsistent linear system (the last equation being $0=-22$).

c) The zero vector is in any span:

$$\begin{pmatrix}0\\0\\0\end{pmatrix} = 0\begin{pmatrix}1\\-1\\2\end{pmatrix} + 0\begin{pmatrix}6\\2\\-1\end{pmatrix}.$$