

### Math 1553 Worksheet 3

September 9, 2016

The professor in your widgets and gizmos class is trying to decide between three different grading schemes for computing your final course grade. The schemes are based on homework (HW), quiz grades (Q), midterms (M), and a final exam (F). The three schemes can be described by the following matrix  $A$ :

$$\begin{array}{l} \text{Scheme 1} \\ \text{Scheme 2} \\ \text{Scheme 3} \end{array} \begin{array}{c} \text{HW} \quad \text{Q} \quad \text{M} \quad \text{F} \\ \left( \begin{array}{cccc} 0.1 & 0.1 & 0.5 & 0.3 \\ 0.1 & 0.1 & 0.4 & 0.4 \\ 0.1 & 0.1 & 0.6 & 0.2 \end{array} \right) \end{array}$$

1. Suppose that you have a score of  $x_1$  on homework,  $x_2$  on quizzes,  $x_3$  on midterms, and  $x_4$  on the final, with potential final course grades of  $b_1, b_2, b_3$ . Write a matrix equation  $Ax = b$  to relate your final grades to your scores.
2. Suppose that you end up with averages of 90% on the homework, 90% on quizzes, 85% on midterms, and a 95% score on the final exam. Use Problem 1 to determine which grading scheme leaves you with the highest overall course grade.

3. a) Keeping  $b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$  as a general vector, write the augmented matrix  $(A \mid b)$ .

b) Row reduce this matrix until you reach row echelon form.

c) Looking at the final matrix in (b), what equation in terms of  $b_1, b_2, b_3$  must be satisfied in order for  $Ax = b$  to have a solution?

d) The answer to (c) also defines the span of the columns of  $A$ . Describe the span geometrically.

e) Solve the equation in (c) for  $b_1$ . Looking at this equation, is it possible for  $b_1$  to be the largest of  $b_1, b_2, b_3$ ? In other words, is it ever possible for the grade under Scheme 1 to be the highest of the three final course grades? Why or why not? Which scheme would you argue for?