Math 1553 Quiz 4

Solutions

1. **a)** [3 points] Give the parametric vector form of the solution to the following system of equations.

$$\begin{aligned} x + \ 3y - 2z &= \ 4 \\ 2x + \ 6y - 2z &= \ 6 \\ 5x + 15y - 8z &= 18 \end{aligned}$$

b) [2 points] Give the parametric vector form of the solution to the following system of equations, without doing any more work.

$$\begin{aligned} x + \ 3y - 2z &= 0 \\ 2x + \ 6y - 2z &= 0 \\ 5x + 15y - 8z &= 0 \end{aligned}$$

Solution.

a) To find the parametric vector form, first we find the parametric form of the solutions by row reduction:

$$\left(\begin{array}{ccc|c} 1 & 3 & -2 & 4 \\ 2 & 6 & -2 & 6 \\ 5 & 15 & -8 & 18 \end{array}\right) \rightsquigarrow \left(\begin{array}{ccc|c} 1 & 3 & 0 & 2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right) \rightsquigarrow \begin{cases} x = -3y + \ 2 \\ y = \quad\ y \\ z = \qquad\quad -1 \end{cases}$$

This gives the parametric vector form

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = y \begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}.$$

b) The solution to the inhomogeneous equations is obtained from the solution to the homogeneous equations by adding a specific solution. So we can get the solution to the homogeneous equations by subtracting the specific solution:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = y \begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix}.$$

2. [1 point each] Which of the following sets of vectors are linearly independent?

a) $\left\{\begin{pmatrix}1\\2\end{pmatrix},\begin{pmatrix}1\\3\end{pmatrix},\begin{pmatrix}2\\3\end{pmatrix}\right\}$ **b)** $\left\{\begin{pmatrix}1\\2\\4\end{pmatrix},\begin{pmatrix}0\\0\\0\end{pmatrix},\begin{pmatrix}-1\\3\\4\end{pmatrix}\right\}$ **c)** $\left\{\begin{pmatrix}1\\2\\5\end{pmatrix},\begin{pmatrix}3\\6\\15\end{pmatrix},\begin{pmatrix}-2\\-2\\-8\end{pmatrix}\right\}$

d) $\left\{\begin{pmatrix}1\\0\\0\end{pmatrix},\begin{pmatrix}1\\1\\0\end{pmatrix}\right\}$ **e)** $\left\{\begin{pmatrix}1\\-1\\2\end{pmatrix},\begin{pmatrix}7\\-2\\3\end{pmatrix},\begin{pmatrix}-2\\2\\-4\end{pmatrix}\right\}$

Solution.

a) These are linearly dependent, because there are more vectors than entries in the vector.

b) These are linearly dependent, because the set contains the zero vector.

c) These are linearly dependent: the same row reduction as in Problem 1 gives

$$\begin{pmatrix}1&3&-2\\2&6&-2\\5&15&-8\end{pmatrix}\rightsquigarrow\begin{pmatrix}1&3&0\\0&0&1\\0&0&0\end{pmatrix},$$

so the second column does not have a pivot.

d) These are linearly independent:

$$x\begin{pmatrix}1\\0\\0\end{pmatrix}+y\begin{pmatrix}1\\1\\0\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}\implies\begin{cases}x+y=0\\ \quad\;\; y=0\end{cases}\implies\begin{cases}x=0\\y=0.\end{cases}$$

e) These are linearly dependent: the third vector is a multiple of the first.