Math 1553 Quiz 5

Solutions

1. [5 points] Find bases for the column space and the null space of the matrix

$$A = \begin{pmatrix} 0 & -4 & 8 & -4 \\ -2 & -2 & 3 & -2 \\ -4 & -12 & 22 & -12 \end{pmatrix}.$$

Solution.

First we row reduce:

$$A \rightsquigarrow \begin{pmatrix} 1 & 0 & \frac{1}{2} & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

The first two columns are pivot columns, so a basis for the column space is

$$\left\{ \begin{pmatrix} 0 \\ -2 \\ -4 \end{pmatrix}, \begin{pmatrix} -4 \\ -2 \\ -12 \end{pmatrix} \right\}.$$

The variables x_3 and x_4 are free; the vector parametric form for the general solution to $Ax = 0$ is

$$x = x_3 \begin{pmatrix} -\frac{1}{2} \\ 2 \\ 1 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \end{pmatrix}.$$

A basis for the null space is

$$\left\{ \begin{pmatrix} -\frac{1}{2} \\ 2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \end{pmatrix} \right\}.$$

2. [5 points] Let V be the subspace with basis

$$\mathcal{B} = \left\{ \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}, \begin{pmatrix} 4 \\ 0 \\ -5 \end{pmatrix} \right\}.$$

The vector

$$x = \begin{pmatrix} 10 \\ -3 \\ -8 \end{pmatrix}$$

is in V. Find $[x]_{\mathcal{B}}$.

Solution.

We have to solve the vector equation $x = c_1 \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} + c_2 \begin{pmatrix} 4 \\ 0 \\ -5 \end{pmatrix}$ in the unknowns c_1, c_2. In augmented matrix form, this is

$$\left(\begin{array}{cc|c} 2 & 4 & 10 \\ -1 & 0 & -3 \\ -1 & -5 & -8 \end{array}\right) \xrightarrow{\text{row reduce}} \left(\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{array}\right).$$

Hence $c_1 = 3$ and $c_2 = 1$, so

$$[x]_{\mathcal{B}} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}.$$