Math 1553 Worksheet 7

October 14, 2016

1. Compute the determinant of

$$A = \begin{pmatrix} 4 & 0 & 0 & 5 \\ 1 & 7 & 2 & -5 \\ 3 & 0 & 0 & 0 \\ 8 & 3 & 1 & 7 \end{pmatrix}$$

alternating signs

using cofactor expansions. Expand along the rows or columns that require the least amount of work.

$3\det\begin{vmatrix} 0 & 0 & 5 \\ 7 & 2 & -5 \\ 3 & 1 & 7 \end{vmatrix} - 0\det\begin{vmatrix} 4 & 0 & 5 \\ 1 & 2 & -5 \\ 8 & 1 & 7 \end{vmatrix} + 0\det\begin{vmatrix} 4 & 0 & 5 \\ 1 & 7 & -5 \\ 8 & 3 & 7 \end{vmatrix} - 0\det\begin{vmatrix} 4 & 0 & 0 \\ 1 & 7 & 2 \\ 8 & 3 & 1 \end{vmatrix} = 3\left(\cancel{0\det\begin{vmatrix} 2 & -5 \\ 1 & 7 \end{vmatrix}} - 0\det\begin{vmatrix} 7 & -5 \\ 3 & 7 \end{vmatrix} + 5\det\begin{vmatrix} 7 & 2 \\ 3 & 1 \end{vmatrix}\right)$

$= (3)(5)\left(7\det|1| - 2\det|3|\right) = (3)(5)(7-6) = \boxed{15 = \det A}$

2. Find the inverse of

$$A = \begin{pmatrix} 4 & 1 & 4 \\ 3 & 0 & 2 \\ 0 & 5 & 0 \end{pmatrix}$$

$\det A = -5\left((4)(2) - (4)(3)\right) = 20$

using the formula

$$A^{-1} = \frac{1}{\det A}\begin{pmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{pmatrix} = \boxed{\frac{1}{20}\begin{pmatrix} -10 & 20 & 2 \\ 0 & 0 & 4 \\ 15 & -20 & -3 \end{pmatrix}}$$

$A_{11} = \begin{pmatrix} 0 & 2 \\ 5 & 0 \end{pmatrix}$ $C_{11} = (-1)^{1+1}\det|A_{11}| = -10.$

$A_{21} = \begin{pmatrix} 1 & 4 \\ 5 & 0 \end{pmatrix}$ $C_{21} = (-1)^3\det|A_{21}| = 20$

$A_{31} = \begin{pmatrix} 1 & 4 \\ 0 & 2 \end{pmatrix}$ $C_{31} = (-1)^4\det|A_{31}| = 2.$

$A_{12} = \begin{pmatrix} 3 & 2 \\ 0 & 0 \end{pmatrix}$ $C_{12} = 0.$

$A_{22} = \begin{pmatrix} 4 & 4 \\ 0 & 0 \end{pmatrix}$ $C_{22} = 0$

$A_{32} = \begin{pmatrix} 4 & 4 \\ 3 & 2 \end{pmatrix}$ $C_{32} = (-1)^5\det|A_{32}| = 4$

$A_{13} = \begin{pmatrix} 3 & 0 \\ 0 & 5 \end{pmatrix}$ $C_{13} = (-1)^4\det|A_{13}| = 15$

$A_{23} = \begin{pmatrix} 4 & 1 \\ 0 & 5 \end{pmatrix}$ $C_{23} = (-1)^5\det|A_{23}| = -20$

$A_{33} = \begin{pmatrix} 4 & 1 \\ 3 & 0 \end{pmatrix}$ $C_{33} = (-1)^6\det|A_{33}| = -3$

3. **a)** Using cofactor expansion, explain why $\det A = 0$ if A has a row or a column of zeros.

(ex) $\begin{pmatrix} a & b & c \\ 0 & 0 & 0 \\ d & e & f \end{pmatrix} \to 0\det|C_{12}| + 0\det|C_{22}| - 0\det|C_{32}|$

(ex) $\begin{pmatrix} a & 0 & d \\ b & 0 & e \\ c & 0 & f \end{pmatrix}$ $\cancel{a\det\begin{vmatrix} 0 & e \\ 0 & f \end{vmatrix}} - 0\det\begin{vmatrix} b & e \\ c & f \end{vmatrix} + \cancel{d\det\begin{vmatrix} b & 0 \\ c & 0 \end{vmatrix}} = 0$

0 0

b) Using cofactor expansion, explain why $\det A = 0$ if A has adjacent identical columns.

(ex) $\begin{pmatrix} a & a & d \\ b & b & e \\ c & c & f \end{pmatrix}$ $a(bf-ec) - a(bf-ec) + d(\underbrace{bc-bc}_{0}) = 0$

true for any values of a, b, c, d, e, f.