Math 1553 Quiz 6

Solutions

1. [2 points each] Compute the determinants of the following matrices.
[Hint: none require extensive calculations.]

a) $\begin{pmatrix} 1 & 1 & 2 \\ -3 & 0 & 1 \\ 1 & 2 & 3 \end{pmatrix}$

Solution.

For this matrix, it is probably easiest to use the formula for the determinant of a 3×3 matrix:

$$\begin{aligned} \det\begin{pmatrix} 1 & 1 & 2 \\ -3 & 0 & 1 \\ 1 & 2 & 3 \end{pmatrix} &= 1\cdot 0\cdot 3 + 1\cdot 1\cdot 1 + 2\cdot(-3)\cdot 2 \\ &\quad - 1\cdot 1\cdot 2 - 1\cdot(-3)\cdot 3 - 2\cdot 0\cdot 1 \\ &= -4 \end{aligned}$$

a) $\begin{pmatrix} 0 & -4 & 0 & 5 \\ 3 & 2 & 1 & 0 \\ 2 & -1 & 0 & 1 \\ 0 & 1 & 0 & 2 \end{pmatrix}$

Solution.

Here it is easiest to use cofactor expansion along the third column, then the first column:

$$\det\begin{pmatrix} 0 & -4 & 0 & 5 \\ 3 & 2 & 1 & 0 \\ 2 & -1 & 0 & 1 \\ 0 & 1 & 0 & 2 \end{pmatrix} = -\det\begin{pmatrix} 0 & -4 & 5 \\ 2 & -1 & 1 \\ 0 & 1 & 2 \end{pmatrix} = +2\det\begin{pmatrix} -4 & 5 \\ 1 & 2 \end{pmatrix} = -26$$

a) $\begin{pmatrix} 3 & 17 & -25 & 4 & 2 \\ 0 & -1 & 37 & -2 & 18 \\ 0 & 0 & 2 & 101 & -32 \\ 0 & 0 & 0 & 2 & -61 \\ 0 & 0 & 0 & 0 & 4 \end{pmatrix}$

Solution.

This is an upper-triangular matrix, so its determinant is the product of the diagonal entries:

$$\det\begin{pmatrix} 3 & 17 & -25 & 4 & 2 \\ 0 & -1 & 37 & -2 & 18 \\ 0 & 0 & 2 & 101 & -32 \\ 0 & 0 & 0 & 2 & -61 \\ 0 & 0 & 0 & 0 & 4 \end{pmatrix} = 3\cdot(-1)\cdot 2\cdot 2\cdot 4 = -48$$

a) $\begin{pmatrix} 1 & 2 & 20 & 7 & 4 & 32 \\ 2 & 4 & 11 & 14 & 17 & 5 \\ -1 & -2 & 37 & 19 & 24 & 2 \\ 3 & 6 & 7 & 8 & 9 & 10 \\ 0 & 0 & 34 & 2 & 7 & 55 \\ -2 & -4 & 5 & 77 & 19 & 0 \end{pmatrix}$

Solution.

The second column is a multiple of the first. Hence the columns are linearly dependent, so the matrix is not invertible, so its determinant is zero.

a) $\begin{pmatrix} d & e & f \\ 2a+3g & 2b+3h & 2c+3i \\ g & h & i \end{pmatrix}^3$, assuming $\det\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = 1$.

(Notice the first matrix is cubed.)

Solution.

The matrix $\begin{pmatrix} d & e & f \\ 2a+3g & 2b+3h & 2c+3i \\ g & h & i \end{pmatrix}$ is obtained from $\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$ by doing one row swap, multiplying one row by 2, and doing one row replacement (not necessarily in that order). Hence

$$\det\begin{pmatrix} d & e & f \\ 2a+3g & 2b+3h & 2c+3i \\ g & h & i \end{pmatrix} = (-1)\cdot 2\cdot \det\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = -2.$$

Cubing a matrix cubes its determinant, so

$$\det\begin{pmatrix} d & e & f \\ 2a+3g & 2b+3h & 2c+3i \\ g & h & i \end{pmatrix}^3 = -8.$$