Math 1553 Worksheet 8

October 21, 2016

1. Let $A = \begin{pmatrix} 2 & -8 & 6 & 8 \\ 3 & -9 & 5 & 10 \\ -3 & 0 & 1 & -2 \\ 1 & -4 & 0 & 6 \end{pmatrix}$.

a) Compute det(A) using row reduction.

$$\xrightarrow{\frac{1}{2}R_1} \begin{pmatrix} 1 & -4 & 3 & 4 \\ 3 & -9 & 5 & 10 \\ -3 & 0 & 1 & -2 \\ 1 & -4 & 0 & 6 \end{pmatrix} \xrightarrow[R_4 = R_1 - R_4]{R_2 = -3R_1 + R_2,\ R_3 = 3R_1 + R_3} \begin{pmatrix} 1 & -4 & 3 & 4 \\ 0 & 3 & -4 & -2 \\ 0 & -12 & 10 & 10 \\ 0 & 0 & -3 & 2 \end{pmatrix} \xrightarrow{\frac{1}{3}R_2} \begin{pmatrix} 1 & -4 & 3 & 4 \\ 0 & 1 & -4/3 & -2/3 \\ 0 & -12 & 10 & 10 \\ 0 & 0 & -3 & 2 \end{pmatrix} \xrightarrow{R_3 = 12R_2 + R_3} \begin{pmatrix} 1 & -4 & 3 & 4 \\ 0 & 1 & -4/3 & -2/3 \\ 0 & 0 & -6 & 2 \\ 0 & 0 & -3 & 2 \end{pmatrix} \xrightarrow{R_4 = -\frac{1}{2}R_3 + R_4} \begin{pmatrix} 1 & -4 & 3 & 4 \\ 0 & 1 & -4/3 & -2/3 \\ 0 & 0 & -6 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix} = B$$

$E_7 \cdots E_2 E_1 A = B$

$\det(E_7) \cdots \det(E_1) \det(A) = \det(B)$ — since B is triangular, det(B) = multiple of diagonals

$(1)(1)\left(\frac{1}{3}\right)(1)(1)(1)\left(\frac{1}{2}\right)\det(A) = (1)(1)(-6)(1)$

$\det(A) = (-6)(3)(2) = \boxed{-36}$

b) Compute det$((A^T)^5)$ without doing any more work.

Property: $\det(A^T) = \det(A) \Rightarrow \det((A)^5) \Rightarrow \det(A \cdot A \cdot A \cdot A \cdot A) = \det(A)^5 = \boxed{(-36)^5}$

c) Compute det(A^{-1}) without doing any more work.

Property: $\det(A^{-1}) = \dfrac{1}{\det(A)} = \boxed{\dfrac{1}{-36}}$

2. Sing the eigenvector song: ♫ *an eigenvector is a v where A times v is λv.* ♪

3. Determine whether the following statements are always true or sometimes false. In the latter case, correct it to make a true statement.

a) A matrix A is not invertible if 0 is an eigenvalue of A.

$\boxed{\text{True}}$ if 0 is an eigenvalue then $Ax = 0x$ has a non-trivial soln which means A has free variables which means A is not invertible

b) If v_1 and v_2 are linearly independent eigenvectors of A, then they must correspond to ~~different eigenvalues~~

$\boxed{\cancel{\text{True}}}$ $\boxed{\text{False}}$ but the converse is true

c) The entries on the main diagonal of A are the eigenvalues of A.

$\boxed{\text{Sometimes False}}$ if A is triangular, then true.

d) The eigenvectors are in the ~~range~~ of the matrix $A - \lambda I$.

$\boxed{\text{False}}$ nullspace

e) The number λ is an eigenvalue of A if and only if there is a nonzero solution to the equation $(A - \lambda I)x = 0$.

$\boxed{\text{True}}$ $Ax - \lambda I x = 0 \Rightarrow Ax = \lambda I x \Rightarrow Ax = \lambda x.$

f) To find the eigenvectors of A, we reduce the matrix ~~A~~ to row echelon form.

$\boxed{\text{False}}$ $(A - \lambda I)$

4. Find a basis for the (-1)-eigenspace of the following matrices.

a) $A = \begin{pmatrix} 2 & 3 & 1 \\ 3 & 2 & 4 \\ 0 & 0 & -1 \end{pmatrix}$

$Av = \lambda v = \lambda I v$ where I is identity matrix $n \times n$ when $v \in \mathbb{R}^n$

in this example $I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$Av - \lambda I v = 0 \Rightarrow (A - \lambda I)v = 0$ This is a homogenous eqn to solve for v!!

$\lambda = -1$

$$(A - (-1)I) = \begin{pmatrix} 3 & 3 & 1 \\ 3 & 3 & 4 \\ 0 & 0 & 0 \end{pmatrix} \xrightarrow{RREF} \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$v_1 + v_2 = 0, \quad v_3 = 0 \Rightarrow v_1 = -v_2, \quad v_2 = v_2, \quad v_3 = 0 \Rightarrow \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = v = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} v_2$$

Basis of (-1)-eigenspace: $\left\{ \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \right\}$

b) $A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

$$(A - (-1)I) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \xrightarrow{RREF} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow$$

$$v_1 + v_2 + v_3 = 0 \Rightarrow v_1 = -v_2 - v_3, \quad v_2 = v_2, \quad v_3 = v_3 \Rightarrow \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = v = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} v_2 + \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} v_3$$

Basis of (-1)-eigenspace: $\left\{ \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \right\}$