Math 1553 Quiz 7

Solutions

1. [5 points] Write a mathematically correct definition of an eigenvector:

> "v is an eigenvector of an $n \times n$ matrix A provided that
>
> $v \neq 0$ and $Av = \lambda v$ for some scalar λ".

2. [5 points] Find all eigenvalues of A, and compute a basis for each eigenspace.

$$A = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$$

Solution.

This is an upper-triangular matrix, so the eigenvalues are the diagonal entries 1 and 3. To find a basis for the 1-eigenspace, we compute

$$A - I = \begin{pmatrix} 0 & 2 \\ 0 & -2 \end{pmatrix} \xrightarrow{\text{rref}} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}.$$

The parametric vector form for the general solution to $(A - I)v = 0$ is $v = x\binom{1}{0}$, so a basis for the 1-eigenspace is $\left\{\binom{1}{0}\right\}$. To find a basis for the 3-eigenspace, we compute

$$A - 3I = \begin{pmatrix} -2 & 2 \\ 0 & 0 \end{pmatrix} \xrightarrow{\text{rref}} \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix}.$$

The parametric vector form for the general solution to $(A - 3I)v = 0$ is $v = y\binom{1}{1}$, so a basis for the 3-eigenspace is $\left\{\binom{1}{1}\right\}$.