MATH 1553-B
MIDTERM EXAMINATION 1

Name		**Section**	

1	2	3	4	5	Total

Please **read all instructions** carefully before beginning.

- Each problem is worth 10 points. The maximum score on this exam is 50 points.
- You have 50 minutes to complete this exam.
- There are no aids of any kind (notes, text, etc.) allowed.
- Please show your work.
- You may cite any theorem proved in class or in the sections we covered in the text.
- Good luck!

Problem 1.

[2 points each]

In this problem, A is an $m \times n$ matrix (m rows and n columns) and b is a vector in $\mathbf{R}^m$. Let $T(x) = Ax$ be the linear transformation associated to A. Circle **T** if the statement is always true (for any choices of A and b) and circle **F** otherwise. Do not assume anything else about A or b except what is stated.

a) **T** **F** If $m \leq n$, then T is onto.

b) **T** **F** If A has fewer than n pivots, then $Ax = b$ has infinitely many solutions.

c) **T** **F** The columns of $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ are linearly independent.

d) **T** **F** If b is in the span of the columns of A, then $Ax = b$ is consistent.

e) **T** **F** The solution set of $Ax = b$ is a span.

Problem 2.

[5 points each]

Acme Widgets, Gizmos, and Doodads has two factories. Factory A makes 10 widgets, 3 gizmos, and 2 doodads every hour, and factory B makes 4 widgets, 1 gizmo, and 1 doodad every hour.

a) If factory A runs for a hours and factory B runs for b hours, how many widgets, gizmos, and doodads are produced? Express your answer as a vector equation.

b) A customer places an order for 16 widgets, 5 gizmos, and 3 doodads. Can Acme fill the order with no widgets, gizmos, or doodads left over? If so, how many hours do the factories run? If not, why not?

Problem 3. [10 points]

Find all values of h such that $\begin{pmatrix} 1 \\ h \\ 5 \end{pmatrix}$ is *not* in the span of $\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 4 \\ 1 \end{pmatrix}$.

Problem 4. [10 points]

Consider the following consistent system of linear equations.

$$\begin{aligned} x_1 + 2x_2 + 3x_3 + 4x_4 &= -2 \\ 3x_1 + 4x_2 + 5x_3 + 6x_4 &= -2 \\ 5x_1 + 6x_2 + 7x_3 + 8x_4 &= -2 \end{aligned}$$

a) [4 points] Find the parametric vector form for the general solution.

b) [3 points] Find the parametric vector form of the corresponding *homogeneous* equations.

c) [3 points] Find a linear dependence relation among the vectors

$$\left\{ \begin{pmatrix} 1 \\ 3 \\ 5 \end{pmatrix}, \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix}, \begin{pmatrix} 3 \\ 5 \\ 7 \end{pmatrix}, \begin{pmatrix} 4 \\ 6 \\ 8 \end{pmatrix} \right\}.$$

Problem 5. [10 points]

Consider the following transformations from $\mathbf{R}^3$ to $\mathbf{R}^2$:

$$T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2x+3y+z \\ 4x+6y+2z \end{pmatrix} \qquad U\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2x+3y+z \\ 4x+6y+2z+2 \end{pmatrix}.$$

a) [3 points] One of these two transformations is *not* linear. Which is it, and why?

b) [3 points] Find the standard matrix for the linear one.

c) [2 points] Draw a picture of the range of the linear one.

d) [2 points] Is the linear one onto? If so, why? If not, find a vector b in $\mathbf{R}^2$ which is not in the range. (It is enough to use the picture in (c).)

[Scratch work]