MATH 1553-B
MIDTERM EXAMINATION 2

Name	

1	2	3	4	5	6	Total

Please **read all instructions** carefully before beginning.

- Each problem is worth 10 points. The maximum score on this exam is 60 points.
- You have 50 minutes to complete this exam.
- There are no aids of any kind (notes, text, etc.) allowed.
- Please show your work.
- You may cite any theorem proved in class or in the sections we covered in the text.
- Good luck!

Problem 1.

[2 points each]

Circle **T** if the statement is always true, and circle **F** otherwise. You do not need to explain your answer.

a) **T** **F** An invertible matrix is a product of elementary matrices.

b) **T** **F** There exists a 3×5 matrix (3 rows, 5 columns) with rank 4.

c) **T** **F** There exists a 3×5 matrix whose null space has dimension 4.

d) **T** **F** If the columns of an $n \times n$ matrix A span $\mathbf{R}^n$, then A is invertible.

e) **T** **F** The solution set of a consistent matrix equation $Ax = b$ is a subspace.

Problem 2. [5 points each]

Let

$$A = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}.$$

a) Find A^{-1}.

b) Solve for x in $Ax = \binom{a}{b}$.

Problem 3.

Let

$$A = \begin{pmatrix} 7 & 1 & 4 & 1 \\ -1 & 0 & 0 & 6 \\ 9 & 0 & 2 & 3 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 0 & 1 & 5 & 4 \\ 1 & -1 & -3 & 0 \\ -1 & 0 & 5 & 4 \\ 3 & -3 & -2 & 5 \end{pmatrix}$$

a) [3 points] Compute $\det(A)$.

b) [3 points] Compute $\det(B)$.

c) [2 points] Compute $\det(AB)$.

d) [2 points] Compute $\det(A^2B^{-1}AB^2)$.

Problem 4.

Consider the following matrix A and its reduced row echelon form:

$$\begin{pmatrix} 2 & 4 & 7 & -16 \\ 3 & 6 & -1 & -1 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 & 2 & 0 & -1 \\ 0 & 0 & 1 & -2 \end{pmatrix}.$$

a) [4 points] Find a basis $\mathcal{B}$ for Nul$A$.

b) [2 points each] For each of the following vectors v, decide if v is in Nul$A$, and if so, find $[x]_{\mathcal{B}}$:

$$\begin{pmatrix} 7 \\ 3 \\ 1 \\ 2 \end{pmatrix} \quad \begin{pmatrix} -5 \\ 2 \\ -2 \\ -1 \end{pmatrix} \quad \begin{pmatrix} -1 \\ 1 \\ 2 \\ 1 \end{pmatrix}$$

Problem 5.

Consider the matrix A and its reduced row echelon form from the previous problem:

$$\begin{pmatrix} 2 & 4 & 7 & -16 \\ 3 & 6 & -1 & -1 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 & 2 & 0 & -1 \\ 0 & 0 & 1 & -2 \end{pmatrix}.$$

a) [4 points] Find a basis for $\operatorname{Col} A$.

b) [3 points] What are $\operatorname{rank} A$ and $\dim \operatorname{Nul} A$?

c) [3 points] Find a different basis for $\operatorname{Col} A$. (Reordering your answer from (a) does not count.) Justify your answer.

Problem 6.

[2 points each]

Which of the following are subspaces of $\mathbf{R}^4$ and **why?**

a) $\text{Span}\left\{\begin{pmatrix}1\\0\\3\\2\end{pmatrix}, \begin{pmatrix}-2\\7\\9\\13\end{pmatrix}, \begin{pmatrix}144\\0\\0\\1\end{pmatrix}\right\}$

b) $\text{Nul}\begin{pmatrix}2 & -1 & 3\\0 & 0 & 4\\6 & -4 & 2\\-9 & 3 & 4\end{pmatrix}$

c) $\text{Col}\begin{pmatrix}2 & -1 & 3\\0 & 0 & 4\\6 & -4 & 2\\-9 & 3 & 4\end{pmatrix}$

d) $V = \left\{\begin{pmatrix}x\\y\\z\\w\end{pmatrix} \text{ in } \mathbf{R}^4 \;\middle|\; xy = zw\right\}$

e) The range of a linear transformation with codomain $\mathbf{R}^4$.

[Scratch work]