MATH 1553-B
PRACTICE MIDTERM 3

Name		**Section**	

1	2	3	4	5	Total

Please **read all instructions** carefully before beginning.

- Each problem is worth 10 points. The maximum score on this exam is 50 points.
- You have 50 minutes to complete this exam.
- There are no aids of any kind (notes, text, etc.) allowed.
- Please show your work.
- You may cite any theorem proved in class or in the sections we covered in the text.
- Good luck!

This is a practice exam. It is similar in format, length, and difficulty to the real exam. It is not meant as a comprehensive list of study problems. I recommend completing the practice exam in 50 minutes, without notes or distractions.

Problem 1.

[2 points each]

In this problem, if the statement is always true, circle **T**; if it is always false, circle **F**; if it is sometimes true and sometimes false, circle **M**.

a) **T** **F** **M** If A is a 3×3 matrix with characteristic polynomial $-\lambda^3 + \lambda^2 + \lambda$, then A is invertible.

b) **T** **F** **M** A 3×3 matrix with two distinct eigenvalues is diagonalizable.

c) **T** **F** **M** If A is diagonalizable and B is similar to A, then B is diagonalizable.

d) **T** **F** **M** A diagonalizable $n \times n$ matrix admits n linearly independent eigenvectors.

e) **T** **F** **M** A stochastic matrix admits a unique steady state.

Problem 2.

[2 points each]

Give an example of a 2×2 real-valued matrix A with each of the following properties. You need not explain your answer.

a) A has no real eigenvalues.

b) A has eigenvalues 1 and 2.

c) A is invertible but not diagonalizable.

d) A is diagonalizable but not invertible.

e) A is positive stochastic.

Problem 3.

Consider the matrix

$$A = \begin{pmatrix} 4 & 2 & -4 \\ 0 & 2 & 0 \\ 2 & 2 & -2 \end{pmatrix}.$$

a) [4 points] Find the eigenvalues of A, and compute their algebraic multiplicities.

b) [4 points] For each eigenvalue of A, find a basis for the corresponding eigenspace.

c) [2 points] Is A diagonalizable? If so, find an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$. If not, why not?

Problem 4.

Consider the matrix

$$A = \begin{pmatrix} 3 & -5 \\ 2 & -3 \end{pmatrix}.$$

a) [3 points] Find the (complex) eigenvalues of A.

b) [2 points] For each eigenvalue of A, find a corresponding eigenvector.

c) [3 points] Find a rotation-scaling matrix C that is similar to A.

d) [1 point] By what factor does C scale?

e) [1 point] By what angle does C rotate?

Problem 5.

Consider the sequence of numbers $0, 1, 5, 31, 185, \ldots$ given by the recursive formula

$$\begin{aligned} a_0 &= 0 \\ a_1 &= 1 \\ a_n &= 5a_{n-1} + 6a_{n-2} \qquad (n \geq 2). \end{aligned}$$

a) [2 points] Find a matrix A such that

$$A\begin{pmatrix} a_{n-2} \\ a_{n-1} \end{pmatrix} = \begin{pmatrix} a_{n-1} \\ a_n \end{pmatrix}$$

for all $n \geq 2$.

b) [3 points] Find an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$.

c) [3 points] Give a formula for A^n. Your answer should be a single matrix whose entries depend only on n.

d) [2 points] Give a non-recursive formula for a_n.

(For extra practice: what happens if you try to use this method to get a closed form for the nth Fibonacci number?)

[Scratch work]